An open Python implementation of FRB/US[*]

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Abstract

FRB/US is the Federal Reserve Board's large-scale macroeconometric model of the United States, used by Board staff for forecasting and for monetary and fiscal policy analysis. The Board publishes the model's full equation system, coefficients, quarterly database and a Python reference implementation (pyfrbus) in the public domain, but the reference package depends on pinned, ageing scientific-Python libraries. This paper documents an open, from-scratch, modern Python reimplementation of FRB/US with VAR (backward-looking) expectations, built directly from the Fed's published `model.xml` and LONGBASE database as part of the PolicyEngine Macro programme. The implementation parses the 284-equation simultaneous system, constructs a sparse analytic Jacobian by symbolic differentiation, and solves each quarter with a damped Newton method using sparse LU factorisation. Validation is exact in the two senses that matter: after add-factoring, the solved baseline reproduces the LONGBASE trajectory for all 284 endogenous variables to machine precision (5.6×10^{-17}), and an identical 100-basis-point federal funds rate shock run in both this implementation and the Fed's own pyfrbus agrees across all endogenous variables and quarters to a maximum absolute difference of 6.0×10^{-9} against the fixed pyfrbus 1.0.0 regression reference and 1.4×10^{-8} against pyfrbus 1.1.1. The simulated policy experiments reproduce the qualitative and quantitative properties of published FRB/US simulations: output falls about half a per cent roughly two years after a funds-rate shock before returning to baseline over ten years; the first-year government-purchases multiplier is 0.72 when the inertial Taylor rule responds, rising to essentially one by the second year when the funds rate is pegged to its baseline path; and an equal-cost personal-tax-rate cut yields multipliers of only 0.2–0.3, inside the published FRB/US and cross-model ranges. A comparison section situates these responses against the recent FRB/US literature (the linearised LINVER benchmark, the framework-review makeup-strategy papers) and against other U.S. benchmarks — the New York Fed DSGE model, the VAR evidence surveyed by Ramey, and CBO's published multiplier ranges — and a cost-push experiment traces the model's answer to a tariff-style price shock. The paper also documents the model in depth — the polynomial-adjustment-cost decision rules, the VAR-expectations closure, the price–wage, spending, labour and financial blocks, and the full menu of policy rules — and the implementation's measured Jacobian sparsity, block structure and performance (about four milliseconds per solved quarter).

Keywords: FRB/US, macroeconometric models, monetary policy, fiscal policy, open-source software, model validation

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1 Introduction

FRB/US is the Federal Reserve Board's large-scale quarterly macroeconometric model of the United States. Introduced in 1996 as the successor to the MPS model, it is one of the principal models that Board staff consult for forecasting and for the analysis of monetary and fiscal policy issues (Brayton and Tinsley, 1996; Brayton et al., 2014). Unusually among central-bank workhorse models, FRB/US is public: the Board publishes the complete equation system with estimated coefficients (`model.xml`), the quarterly historical-plus-projection database (`LONGBASE.TXT`), extensive technical documentation, and reference simulation platforms in EViews and, since 2021, in Python (the `pyfrbus` package), all in the public domain (Board of Governors of the Federal Reserve System, 2026b,a).

This paper documents `frbus`, an open, from-scratch reimplementation of the FRB/US simulation platform in modern Python, developed as part of PolicyEngine Macro — PolicyEngine's programme of open macroeconomic modelling infrastructure that complements its tax–benefit microsimulation models. The motivation is practical. The Fed's own `pyfrbus` reference package is an invaluable specification of the model's semantics, but it is pinned to an ageing scientific-Python stack (`sympy` 1.3, `scikit-umfpack`, pre-2.0 `pandas`) and does not install cleanly on current interpreters without patching. A dependable, continuously tested implementation on current `numpy`/`scipy`/`sympy`/`pandas` lowers the barrier to using FRB/US in research and in downstream policy tooling, where microsimulation-based revenue and distributional estimates can be combined with model-consistent macroeconomic feedbacks.

The reimplementation is deliberately conservative: it treats the Fed's published `model.xml` and LONGBASE data as the single source of truth and follows `pyfrbus` semantics exactly, while making independent numerical choices where these are equivalence-preserving (a single-block Newton solve of the full simultaneous system per period, rather than the vendor's block decomposition). Fidelity is then demonstrated rather than asserted, in two layers. First, a *tracking invariant*: after computing add factors, solving the model over 2026Q1–2030Q4 reproduces the LONGBASE baseline for all 284 endogenous variables to 5.6×10^{-17} — machine precision. Second, *cross-validation*: the identical monetary-policy experiment agrees across all 284 endogenous variables and all 20 quarters to a maximum absolute difference of 6.0×10^{-9} against the fixed `pyfrbus` 1.0.0 regression reference and 1.4×10^{-8} against `pyfrbus` 1.1.1. The fixed-reference comparison runs as a hard gate in continuous integration; Section 5 reports the version-to-version check.

Beyond replication of the reference implementation, the model's simulation properties line up with what the Fed has published about FRB/US itself. A 100-basis-point shock to the inertial Taylor rule lowers real GDP by 0.55 per cent at its trough seven quarters out, raises the unemployment rate by up to 0.26 percentage point, and lowers core PCE inflation by up to 0.034 percentage point — signs, magnitudes and timing consistent with the funds-rate-shock responses under VAR expectations reported by Brayton et al. (2014); over a 40-quarter horizon the economy returns essentially to baseline. The paper then uses the platform to trace out the model's fiscal-policy menu with three budget-comparable experiments. A government-purchases increase of 1 per cent of GDP for eight quarters yields a first-year multiplier of 0.72 when the inertial Taylor rule responds; re-running the identical shock with the funds rate pegged to its baseline path — a two-line change, thanks to the model's policy-rule switch architecture — raises the multiplier to 0.99 in the second year, quantifying how much of the crowding out is the central bank's doing. An equal-cost personal-tax-rate cut delivers multipliers of only 0.22 in the first year and 0.32 in the second, reflecting the model's split between rule-of-thumb and permanent-income consumers. All three sets of numbers sit inside the ranges published for FRB/US in the multi-model fiscal-stimulus comparison of Coenen et al. (2012) and in the Board's own documentation. A sixth experiment — a four-quarter cost-push shock to the core inflation equation, the model's native rendering of the tariff-style supply shocks of 2025–2026 — rounds out the menu, and a dedicated comparison section places all of the responses alongside the recent FRB/US literature and other U.S. model and VAR benchmarks.

The remainder of the paper proceeds as follows. Section 2 situates FRB/US and this implementation in the literature. Section 3 presents the model in depth: its history, the sector composition of the 284-equation system, a derivation of the polynomial-adjustment-cost (PAC) decision rule and its VAR-expectations closure, walkthroughs of the price–wage, consumption, investment, labour and financial blocks with the coded equations and coefficients, and the full menu of monetary policy rules and fiscal closures. Section 4 describes the implementation: the XML schema, lexing and index substitution, the symbolic sparse Jacobian and its measured sparsity, the structural block anatomy of the system, the damped Newton solver, and performance benchmarks. Section 5 presents the validation evidence. Section 6 presents the six simulation experiments. Section 7 consolidates the comparisons with the Fed's reference implementation, with published FRB/US simulations, and with other U.S. model benchmarks. Section 8 discusses limitations. Appendices provide a variable glossary, caveats on deterministic simulation, and a reproduction recipe.

2 Related literature

The FRB/US lineage. FRB/US descends from a half-century of structural modelling at the Federal Reserve Board. Its predecessor, the MIT–Penn–SSRC (MPS) model, served as the Board's workhorse from around 1970; Brayton et al. (1997a) recount how accumulating dissatisfaction with the MPS treatment of expectations and dynamics — essentially unrestricted distributed lags, exposed to the Lucas critique — motivated the 1996 redesign. FRB/US's two signature choices date from that redesign. First, dynamics are derived from explicit adjustment-cost optimisation: the polynomial-adjustment-cost framework of Tinsley (1993, 2002) generalises quadratic adjustment costs to higher-order frictions, delivering estimated error-correction decision rules that remain interpretable as optimising behaviour (Section 3.3). Second, expectations are explicit and switchable: every behavioural equation separates its expectational inputs into named variables that can be generated either by small estimated VARs or model-consistently, so the same estimated structure supports both boundedly rational and rational-expectations analysis (Brayton et al., 1997b). Brayton et al. (2014) is the standard modern summary and the source of the published impulse responses used for comparison in Section 6.

Policy analysis with FRB/US. The model has been the vehicle for a long line of influential policy research. Reifschneider and Williams (2000) used stochastic simulations of FRB/US to show that the zero lower bound becomes quantitatively important for inflation targets around or below 2 per cent — under a Taylor rule with a 2 per cent target the funds rate is constrained roughly a tenth of the time — and proposed the "lower-for-longer" compensating rules that later underpinned makeup-strategy analysis. Laubach and Williams (2003), estimating the natural real rate jointly with potential output, supplied the r^* concept that enters FRB/US policy rules as `rstar` and conditions its low-rate scenario work. Reifschneider (2016) used FRB/US to ask whether asset purchases and forward guidance could substitute for constrained rate cuts in a future recession, and Bernanke et al. (2019) ran large stochastic-simulation comparisons of Taylor, Reifschneider–Williams and temporary price-level-targeting rules in FRB/US under both expectations options, finding makeup strategies robust to imperfect credibility. The model remained central to the Federal Reserve's 2019–2020 framework review — Hebden et al. (2020) stress-test average-inflation-targeting and price-level-targeting strategies in FRB/US across its full expectations menu, finding that under VAR expectations makeup strategies provide no early stabilisation and later overheat the economy — and to its aftermath: Brayton and Reifschneider (2022) distil FRB/US into a linearised version (LINVER) for large-scale stochastic ELB simulations, and Kiley (2025) uses FRB/US to quantify how incorporating long-run inflation expectations into policy rules affects anchoring, the most recent Board research use of the model at the time of writing. On the fiscal side, FRB/US was one of the models in the multi-country fiscal-stimulus comparison of Coenen et al. (2012), which reports first-year U.S. government-consumption multipliers of 0.7–1.0 without monetary accommodation (FRB/US near the bottom of that range, rising to roughly 1.1–1.2 with two years of accommodation) and labour-tax-cut multipliers of only 0.2–0.4; Cashin et al. (2018) construct a FRB/US-based "fiscal effect" measure of discretionary policy around the Great Recession, with personal-tax MPCs of a few tenths. These published magnitudes are the yardsticks for the experiments of Section 6, alongside the survey of multiplier evidence in Whalen and Reichling (2015).

Open-source macro modelling. FRB/US is unusual among central-bank workhorse models in being fully public: equations, coefficients, database, documentation and simulation code are all in the public domain (Board of Governors of the Federal Reserve System, 2026b). The Board's own `pyfrbus` package (Board of Governors of the Federal Reserve System, 2026a) moved the platform from EViews to Python in 2021, substantially widening access; but, as Section 4 details, it is pinned to a scientific-Python stack of that era. This paper's contribution sits in the small but growing space of independently engineered, continuously tested open implementations of official policy models — the macro counterpart to open tax–benefit microsimulation platforms such as PolicyEngine's, with which it shares tooling conventions (public artefacts as the single source of truth, validation gates in continuous integration, semantic versioning of data vintages). The intended payoff is composability: microsimulation-based revenue and distributional estimates on one side, and model-consistent macroeconomic feedbacks from a published, Fed-maintained structure on the other.

3 The FRB/US model

3.1 History and design

FRB/US was introduced at the Federal Reserve Board in 1996, replacing the MPS model that had been in use since the 1970s (Brayton and Tinsley, 1996; Brayton et al., 1997a). Its design responded to the Lucas-critique era: most behavioural equations are derived from explicit optimisation problems of households, firms and financial-market participants, with expectations appearing explicitly as separate, switchable variables (Brayton et al., 1997b), while retaining the empirical fit and sectoral detail expected of a staff forecasting model. The current public model file specifies 367 variables, of which 284 are endogenous — 120 estimated behavioural equations and 164 identities — while the remaining 83 are exogenous drivers, policy instruments and switches (Board of Governors of the Federal Reserve System, 2026a).

3.2 Structure: the 284 equations by sector

The `model.xml` file tags every variable with a sector and an equation type. Table 1 reports the composition of the endogenous block, counted directly from the parse output of the implementation described in Section 4.

Table 1: The 284 endogenous equations of FRB/US by sector (April 2026 `model.xml`)

Sector	Behavioural	Identity	Total	Representative variables
Wages and prices	23	20	43	`picxfe`, `pieci`, `pipxnc`, `ptr`
Financial	23	13	36	`rff`, `rg10`, `rbbb`, `req`, `wpsn`
Government	19	14	33	`egfe`, `trp`, `tpn`, `gfdbtn`
Nominal income	3	30	33	`ypn`, `yhln`, `ynicpn`
Expectations	5	27	32	`zeco`, `zpicxfe`, `zrff10`, `zgap05`
Aggregate output	9	13	22	`xgdp`, `xfs`, `xgap2`, `xb`
Foreign trade	6	15	21	`ex`, `emo`, `fpxr`
Labor market	9	10	19	`lhp`, `lww`, `leh`, `lur`, `lfpr`
Household expenditures	7	11	18	`eco`, `ecd`, `eh`
Business expenditures	6	8	14	`ebfi`, `ki`, `rtbfi`
Foreign activity	10	3	13	`fgdp`, `fpic`, `frs10`
Total	120	164	284	

Counts from the implementation's parse of `model.xml` (sector and `equation_type` tags). A further 83 variables are exogenous: policy settings (`pitarg`, `trptx`, `rfffix`, `rffmin`), switches (`dmp*`, `dfp*`), trends and conversion factors.

The expenditure side covers household consumption of non-durables and services (`eco`) and durables (`ecd`); residential investment (`eh`); business fixed investment, consolidated in the current vintage into a single chain-weighted aggregate (`ebfi`) with an inventory block; government purchases at federal and state–local level; and foreign trade. The income side disaggregates labour, transfer and property income, the latter defined more broadly than in the NIPAs (households "pierce the corporate veil," counting retained earnings, and inflation-adjust their interest income). The price–wage block is built around two structural inflation equations linked by trend unit labour costs; the labour block maps hours into employment and unemployment through an Okun's-law chain; and the financial block maps the federal funds rate into the term structure, corporate rates, equity values and household wealth. Sections 3.5–3.9 walk through these blocks; we first develop the adjustment-cost machinery they share.

3.3 Polynomial adjustment costs: from cost function to decision rule

The dynamics of most key non-financial variables are governed by the generalised adjustment-cost framework known as PAC — polynomial adjustment costs (Tinsley, 1993, 2002). This subsection derives the decision rule as coded, following the Board's technical notes distributed with the model (`PAC_in_FRBUS.pdf`, `pac_basics.pdf`).

Cost function and Euler equation. An agent choosing y_t (typically a logarithm) minimises

$$C_t \;=\; \sum_{i=0}^{\infty} \beta^i \Big[\big(y_{t+i} - y^*_{t+i}\big)^2 \;+\; \sum_{k=1}^{m} b_k \big((1-L)^k\, y_{t+i}\big)^2 \Big], \tag{1}$$

which penalises deviations of y from its desired level y^* and movements in the level and $m-1$ higher time-differences of y; $\beta = 0.98$ is a fixed discount factor and L the lag operator (expectation operators are suppressed). The case $m = 1$ is the familiar quadratic-adjustment-cost model; most FRB/US equations have m between 2 and 4. Differentiating with respect to y_t, each friction term contributes $2b_k(1-L)^k(1-\beta F)^k y_t$, where $F \equiv L^{-1}$ is the lead operator, giving the Euler equation

$$\left(y_t - y_t^*\right) \;+\; \sum_{k=1}^{m} b_k \left[(1-L)(1-\beta F)\right]^k y_t \;=\; 0. \tag{2}$$

Factorisation. The operator $1 + \sum_k b_k[(1-L)(1-\beta F)]^k$ is a self-reciprocal polynomial in L and βF (the coefficient on $(\beta F)^k$ equals that on L^k), and Tinsley (1993) shows that any such polynomial factors into lag and lead parts $\hat{A}(L)\hat{A}(\beta F)$. Normalising $A(z) = 1 + \alpha_1 z + \cdots + \alpha_m z^m$ and pinning the constant by the steady-state requirement $y = y^*$ when $L = F = 1$ yields the compact Euler equation

$$A(\beta F)\, A(L)\, y_t \;=\; c\, y_t^*, \qquad c = A(1)\, A(\beta). \tag{3}$$

The lag polynomial $A(L)$ carries the intrinsic (backward) dynamics; the lead polynomial $A(\beta F)$ carries the discounted forward-looking response to the target path.

Companion form. Writing $x_t \equiv A(L)y_t$ and stacking m leads of x in a vector g_t, equation (3) becomes $g_t = f_t + G\, g_{t+1}$ with $f_t = (c\, y_t^*, 0, \ldots, 0)'$ and

$$G \;=\; \begin{bmatrix} -\alpha_1\beta & -\alpha_2\beta^2 & \cdots & -\alpha_{m-1}\beta^{m-1} & -\alpha_m\beta^m \\ & I_{m-1} & & & 0 \end{bmatrix}, \tag{4}$$

so that forward substitution gives $A(L)\, y_t = \xi' \sum_{i=0}^{\infty} G^i\, \xi\, c\, y_{t+i}^*$, with ξ the first-coordinate selector.

Decision rule. Decomposing $A(L)y_t = \Delta y_t + A(1)y_{t-1} - A^*(L)\Delta y_{t-1}$, where $A^*(L)$ collects the tail coefficients $\alpha_k^* = \sum_{j=k+1}^{m} \alpha_j$, and converting the forward level sum into a forward difference sum yields the PAC decision rule

$$\Delta y_t \;=\; a_0\left(y_{t-1}^* - y_{t-1}\right) \;+\; \sum_{k=1}^{m-1} a_k\, \Delta y_{t-k} \;+\; E_{t-1} \sum_{j=0}^{\infty} d_j\, \Delta y_{t+j}^*, \tag{5}$$

with exact coefficient mappings

$$a_0 = d_0 = A(1), \qquad a_k = -\sum_{j=k+1}^{m} \alpha_j, \qquad d_j = A(1) - A(1)A(\beta) \sum_{i=0}^{j-1} \xi' G^i \xi. \tag{6}$$

The change in y error-corrects on the lagged gap to target with speed $a_0 = A(1)$, inherits its own lags from the higher-order frictions, and responds to the discounted expected path of target changes.

Target decomposition and Z variables. In FRB/US the target is decomposed as $y^* = y^{1*} + \gamma\, y^{0*}$: a trending I(1) component y^{1*} (denoted by "Q" variables such as `qeco`, specified before estimation from theory or cointegrating regressions) and a stationary component y^{0*} whose loading γ is estimated inside the PAC equation. The two expected-future sums are assigned to explicit expectations variables,

$$\begin{aligned} \Delta y_t \;&=\; a_0\big(y^{1*}_{t-1} - y_{t-1}\big) \;+\; \sum_{k=1}^{m-1} a_k\, \Delta y_{t-k} \;+\; Z^1_t \;+\; Z^0_t, \\ Z^1_t &= \sum_i d_i\, E\, \Delta y^{1*}_{t+i}, \qquad Z^0_t = \sum_i h_i\, E\, y^{0*}_{t+i}, \end{aligned} \tag{7}$$

with weights $h_i = A(1)A(\beta)\, \iota' G^i \iota$ and $d_i = A(1)A(\beta)\, \iota'[I-G]^{-1}G^i\iota$. On a balanced-growth path with $\Delta y = \Delta y^{1*} = g$, the rule delivers $y = y^{1*}$ only if $\sum_k a_k + \sum_i d_i = 1$; the current vintage imposes this *growth-neutrality restriction* directly in estimation (earlier vintages instead added the correction term $[1 - \sum_k a_k - \sum_i d_i]\, g$ to Z^1). Finally, most spending equations aggregate optimising agents with rule-of-thumb agents whose spending tracks income or output x:

$$\Delta y_t \;=\; \gamma_o\Big(a_0\big(y^{1*}_{t-1} - y_{t-1}\big) + \sum_{k=1}^{m-1} a_k\, \Delta y_{t-k} + Z^1_t + Z^0_t\Big) \;+\; (1-\gamma_o)\, \Delta x_t. \tag{8}$$

PAC governs consumption (`eco`, `ecd`), fixed investment (`eh`, `ebfi`), aggregate hours (`lhp`) and dividends (`ynidn`), and the same error-correction-with-expectations logic organises the price–wage block.

3.4 Expectations: VAR and MCE

FRB/US supports two treatments of the roughly 25 explicit expectations (Z) variables (Brayton et al., 1997b, 2014), selectable individually or in groups (all; asset pricing; wage–price).

VAR expectations. Under the configuration implemented here, expectations are generated by small estimated vector autoregressions. The core VAR contains the funds rate, overall PCE inflation and the output gap, specified in error-correction form around *moving endpoints* — long-run survey expectations of the funds rate and inflation (`rtr`, `ptr`), each a random walk, and a zero endpoint for the gap:

$$\Delta X^c_t \;=\; \Lambda_0\big(X^c_{t-1} - X^{c\infty}_{t-1}\big) \;+\; \sum_{i=1}^{3} \Lambda_i\, \Delta X^c_{t-i}. \tag{9}$$

For each PAC equation, the target components are appended to the core VAR with restrictions forcing long-horizon forecasts to the appropriate endpoints, and the combined four-lag system is written in companion form $z_t = H z_{t-1}$, so $E_{t-1} z_{t+i} = H^{i+1} z_{t-1}$. Substituting into the infinite sums of (7) and collapsing them with Kronecker (vec) algebra yields fixed-coefficient, purely backward-looking expressions

$$Z^0_t = h_0' z_{t-1}, \qquad Z^1_t = h_1' z_{t-1}, \tag{10}$$

$$\begin{aligned} h_0 &= A(1)A(\beta)\big[\iota_m' \otimes H'\big]\big[I - G \otimes H'\big]^{-1}\big[\iota_m \otimes \iota_{h0}\big], \\ h_1 &= A(1)A(\beta)\big[\iota_m'(I_m - G)^{-1} \otimes H'\big]\big[I - G \otimes H'\big]^{-1}\big[\iota_m \otimes \iota_{h1}\big], \end{aligned} \tag{11}$$

where ι_{h0}, ι_{h1} select the target components within z. Because h_0, h_1 are nonlinear functions of the PAC coefficients, β and H, the Board estimates each PAC equation by iterative least squares. Asset-price expectations are present values rather than PAC sums: for example the expected average funds rate over ten years satisfies $Z_t^{pv} = (1-w)\,\iota'[I - wH]^{-1} z_t$ with w set by duration, and endpoint restrictions ensure the coefficients on the funds rate and its endpoint sum to one while those on inflation sum to zero. Asset-price expectations condition on current-quarter information; all other sectors use lagged information only.

Because every expectations term in (10) depends only on current and lagged observables, the VAR version of the model is fully backward-looking: the 284-equation system is simultaneous *within* each quarter but recursive *across* quarters, which is what permits the period-by-period solution strategy of Section 4.

Model-consistent expectations. Under MCE the infinite forward sums are instead collapsed into finite-lead recursions,

$$Z_t^0 \;=\; -\sum_{i=1}^{m} \alpha_i \beta^i \, Z_{t+i}^0 \;+\; A(1)A(\beta)\, y_t^{0*}, \tag{12}$$

(and analogously for Z^1), solved jointly with the model's own future paths by extended-path methods with terminal conditions — the Fed's EViews platform alternates between the backward-looking model and a partial forward model, adjusting add factors on the expectations equations until model-consistent expectation errors vanish. MCE variants are out of scope for this implementation (Section 8).

3.5 The price–wage sector

The nominal core of FRB/US is a pair of structural inflation equations — core PCE price inflation (`picxfe`) and growth of the employment cost index for hourly compensation (`pieci`) — specified as new-Keynesian Phillips curves with indexation, following Cogley–Sbordone, and estimated jointly under model-consistent expectations (Board technical note `price_wage_sector_083013.pdf`). Both take the form

$$\Pi_t \;=\; \beta\, E_{t-1}\Pi_{t+1} \;+\; \kappa\, E_{t-1}\mu_t \;+\; \epsilon_t, \qquad \Pi_t \;\equiv\; \pi_t - \gamma\,\hat{\pi}_t - (1-\gamma)\,\bar{\pi}_t, \tag{13}$$

where $\hat{\pi}$ is a lagged-inflation index, $\bar{\pi}$ trend inflation (the ten-year survey expectation `ptr`, a random walk), μ a markup gap, and $\beta = 0.98$. The price markup gap is a weighted average of the detrended labour share and the import-price/final-sales relative price; the wage markup gap reduces to an unemployment-gap term less the labour share. As coded in the current vintage:

$$\pi_t^p \;=\; \frac{\gamma_p\, \pi_{t-1}^p \;+\; \beta\, Z_t^{\pi p} \;+\; (1-\beta)(1-\gamma_p)\,\bar{\pi}_{t-1} \;+\; \kappa_p \cdot 400\, \ln\!\big(\mathtt{qpcnia}_{t-1}/\mathtt{pcnia}_{t-1}\big)}{1+\gamma_p\beta}, \tag{14}$$

with $\gamma_p = 0.671$ and $\kappa_p = 0.00191$, and

$$\begin{aligned}\pi_t^w \;=\; \Big[&\tfrac{1}{4}\gamma_w\big((1-\beta)(\pi_{t-1}^w+\pi_{t-2}^w+\pi_{t-3}^w) + \pi_{t-4}^w\big) + \beta\, Z_t^{\pi w} + (1-\beta)(1-\gamma_w)\big(\bar{\pi}_{t-1} + g_{t-1}^w\big) \\ &- \alpha_w\big(\mathtt{lur}_{t-1} - \mathtt{lurnat}_{t-1}\big) + \kappa_w \cdot 400\, \ln\!\big(\mathtt{qpl}_{t-1}/\mathtt{pl}_{t-1}\big)\Big] \Big/ \big(1 + \tfrac{1}{4}\gamma_w\beta\big),\end{aligned} \tag{15}$$

with $\gamma_w = 0.685$, $\alpha_w = 0.0167$, $\kappa_w = 0.000957$, and trend wage inflation equal to trend price inflation plus trend productivity growth less the trend drift of the consumption/output relative price ($g^w = \texttt{hlprdt} - 400\,\texttt{huqpct}$). Three features deserve note. First, slack affects prices only *indirectly*: the unemployment gap enters the wage equation, and wages reach prices through the labour-share (unit-labour-cost) error-correction terms $\ln(\texttt{qpcnia}/\texttt{pcnia})$ and $\ln(\texttt{qpl}/\texttt{pl})$ and through expected inflation. Second, the estimated slopes are small — the Board's estimation puts κ_p around 0.003 and imposes $\kappa_w = 0.5\,\kappa_p$ — the flat reduced-form Phillips curve visible in the simulations of Section 6. Third, the NAIRU (`lurnat`) is a random walk, and trend expectations (`ptr`) respond only slowly to realised inflation, so one-off shocks barely move the nominal anchor.

Around this core, the sector's remaining equations are semi-structural: overall PCE inflation adds relative food and energy price movements to `picxfe`; the aggregate non-consumption price (`pipxnc`, covering investment, government purchases and exports) follows consumption inflation adjusted for the trend investment/consumption relative-price drift, with error correction and an exchange-rate pass-through term; and nine component deflators track `pxnc` one-for-one with level adjustments that offset chain-aggregation discrepancies.

3.6 Household consumption

Consumption illustrates the full PAC apparatus. Total desired consumption (`qec`, including the service flow from durables) comes from a cointegrating regression on *permanent income by category* and wealth:

$$\ln \texttt{qec} \;=\; 0.365 \;+\; 0.494\,\ln \texttt{zyh}^l \;+\; 0.186\,\ln \texttt{zyht} \;+\; 0.133\,\ln \texttt{zyhp} \;+\; 0.187\,\ln(\texttt{wpo}+\texttt{wps}), \quad (16)$$

where $\texttt{zyh}^l$, `zyht` and `zyhp` are expected present values of labour, transfer and property income (discounted at 25 per cent annually, so "permanent income" is dominated by a horizon of a few years), and `wpo` and `wps` are real non-equity and stock-market wealth. The propensity ordering — highest out of labour income, then transfers, lowest out of property income, with a distinct wealth channel — is what makes the *composition* of an income change matter in FRB/US: transfers and labour income move spending far more than an equal-sized change in property income. Property income itself is defined more broadly than in the NIPAs: it adds the imputed service flow from durables and corporate retained earnings, and subtracts inflation losses on nominal government debt holdings.

The nondurables-and-services equation is then PAC with $m = 2$ and a rule-of-thumb share of 0.308:

$$\Delta \ln \texttt{eco}_t = \big(0.158\,\ln \tfrac{\texttt{qeco}_{t-1}}{\texttt{eco}_{t-1}} + 0.412\,\Delta \ln \texttt{eco}_{t-1} + Z_t^{eco}\big)(1-0.308) + 0.308\,\Delta \ln(\texttt{yhl}+\texttt{yht})_t, \quad (17)$$

so that roughly a third of spending tracks current labour-plus-transfer income — the model's hand-to-mouth channel, which drives the tax-cut experiment of Section 6. Durables (`ecd`) follow a stock-adjustment PAC whose target converts the desired durables stock into gross investment, with user cost $\texttt{rccd} = \max(100\,\texttt{jrcd} + \texttt{rcar} - \texttt{zpi5},\, 0.01)$ (depreciation plus the real consumer-loan rate) entering with elasticity -0.584, plus a cyclical term in the expected output gap.

3.7 Business and residential investment

Business fixed investment (`ebfi`) combines PAC-optimising firms with an accelerator: a share 0.384 of investment growth simply tracks lagged business output growth (a cash-flow/liquidity channel), while the remainder error-corrects to the target capital-accumulation path:

$$\Delta \ln \texttt{ebfi}_t = \left(0.074 \ln \tfrac{\texttt{qebfi}_{t-1}}{\texttt{ebfi}_{t-1}} + 0.522\,\Delta \ln \texttt{ebfi}_{t-1} + 0.302\,\Delta \ln \texttt{ebfi}_{t-2} + Z_t^{ebfi}\right)(1 - 0.384) \\ + 0.384\left(\Delta \ln \texttt{xb}_{t-1} - \tfrac{\texttt{hgpbfir}_{t-1}}{400}\right). \quad (18)$$

The target embodies a neoclassical desired investment–output ratio with unit user-cost elasticity, $\texttt{vbfi} \propto (\texttt{pkbfir}/\texttt{pbfir})/\texttt{rtbfi}$, where the user cost $\texttt{rtbfi}$ combines depreciation, expected relative-price appreciation, corporate taxation (investment tax credit, depreciation allowances, the corporate rate) and a financing rate that is an even mix of after-tax corporate debt (five-year Treasury plus the BBB spread) and the required return on equity. This is the model's main channel from the term structure and credit spreads to real activity. Residential investment ($\texttt{eh}$) is PAC with $m = 3$ plus a borrowing-cost term in the lagged *change* of the nominal mortgage rate; its target user cost is the after-tax real mortgage rate plus property tax and depreciation. Inventory investment error-corrects the stock–sales ratio to a slowly declining filtered trend.

3.8 The labour block and Okun's law

Employment and unemployment are determined by a chain running from output to hours to bodies. Aggregate business hours ($\texttt{lhp}$) follow a modified PAC in which about 37 per cent of hours adjustment is subject to frictions while the rest tracks current target growth:

$$\Delta \ln \texttt{lhp}_t = 0.202 \ln \tfrac{\texttt{qlhp}_{t-1}}{\texttt{lhp}_{t-1}} + 0.203\,\Delta \ln \texttt{lhp}_{t-1} + 0.372\,Z_t^{lhp} \\ + 0.628\left(\Delta \ln \texttt{xbo}_t - \tfrac{\texttt{hlprdt}_{t-1}}{400}\right) - 0.127\left(\Delta \ln \texttt{xbo}_{t-1} - \tfrac{\texttt{hlprdt}_{t-2}}{400}\right), \quad (19)$$

with target hours equal to adjusted business output divided by trend labour productivity, so that the gap between actual and trend productivity is procyclical. About 30 per cent of any hours change is absorbed by the workweek ($\texttt{lww}$) in the short run, the remainder by employment ($\texttt{lep} = \texttt{lhp}/\texttt{lww}$). Household employment adds government workers and an error-correcting payroll/household discrepancy; labour-force participation ($\texttt{lfpr}$) error-corrects to its trend with a discouraged-worker term in the unemployment gap; and the unemployment rate then follows as $\texttt{lur} = 100\,(1 - \texttt{leh}/\texttt{lf})$ (tagged behavioural in `model.xml`, with an additive error term, but numerically an identity). The resulting reduced-form co-movement between the output gap and the unemployment gap is FRB/US's Okun's law — visible in Section 6, where a one-per-cent output loss is associated with roughly a half-point unemployment increase, building over several quarters.

3.9 The financial block: term structure, equity and wealth

Financial prices are the model's forward-looking transmission channel. Long-term rates are expectations-hypothesis averages of the expected funds-rate path plus counter-cyclical term premia:

$$\texttt{rg10}_t = \texttt{zrff10}_t + \texttt{rg10p}_t, \qquad \texttt{rg10p}_t = 1.05 - 0.461\,\texttt{zgap10}_t + \text{AR}(1)\text{ error}, \quad (20)$$

where under VAR expectations $\texttt{zrff10}$ is the duration-weighted present value $(1 - w)\,\iota'[I - wH]^{-1}z_t$ of expected funds rates, loading on current and lagged funds rates, inflation, the output gap and the endpoint variables. Corporate rates add a counter-cyclical BBB risk premium

($\texttt{rbbb} = \texttt{rg10} + \texttt{rbbbp}$); the required real return on equity is the real 30-year rate plus an equity premium tied to the BBB premium. Stock-market wealth is a Gordon-model valuation of dividends,

$$\ln \texttt{wpsn}_t \;=\; \ln\left(0.5\left(\texttt{ynicpn}_t - \texttt{tcin}_t\right)\right) \;-\; 0.25\left(\texttt{req}_t - \texttt{zdivgr}_t\right) \;+\; \text{const}, \tag{21}$$

so equity values fall when required returns rise relative to expected dividend growth; dividends themselves are PAC-smoothed toward a constant share of after-tax profits. Non-equity property wealth (`wpon`) accumulates personal saving, durables net investment and capital gains, with house prices error-correcting to rents. Through equations (16) and (18)–(21), a funds-rate shock therefore propagates simultaneously through user costs, long rates, equity and housing wealth, and the exchange rate, before feeding back into slack and, weakly, inflation.

3.10 Monetary policy rules

Monetary policy is modular. The effective funds rate is set by

$$\texttt{rff}_t \;=\; \max\left(\texttt{rffrule}_t,\; \texttt{rffmin}_t\right) \tag{22}$$

(as coded, a `@recode` indicator expression, with an optional threshold-based liftoff mechanism controlled by `dmptrsh` and logistic switch variables), where `rffmin` is the effective lower bound (0.125 in the baseline data; set to 0 to impose a strict ZLB) and `rffrule` mixes the available rules through exogenous 0/1 switches:

$$\begin{aligned}\texttt{rffrule}_t = \texttt{dmpex}\,\texttt{rfffix}_t + \texttt{dmprr}\left(\texttt{rrfix}_t + \bar{\pi}_t\right) + \texttt{dmptay}\,\texttt{rfftay}_t \\ + \texttt{dmptlr}\,\texttt{rfftlr}_t + \texttt{dmpintay}\,\texttt{rffintay}_t + \texttt{dmpalt}\,\texttt{rffalt}_t.\end{aligned} \tag{23}$$

Table 2 enumerates the rules with their coefficients as published in `model.xml`; $\bar{\pi}_t$ denotes the four-quarter average of core PCE inflation, $\tilde{x}_t$ the output gap (`xgap2`), and r^*_t the policymakers' perceived equilibrium real rate (`rstar`, exogenous in the experiments here, or converging to the model's real rate when $\texttt{drstar} = 1$).

The default in the Fed's demonstration programs, in the LONGBASE projection and throughout this paper is the inertial Taylor rule,

$$r^{ff}_t \;=\; \rho\, r^{ff}_{t-1} \;+\; (1-\rho)\Big[r^*_t + \bar{\pi}_t + 0.5\left(\bar{\pi}_t - \pi^{targ}_t\right) + 1.0\,\tilde{x}_t\Big] \;+\; \varepsilon_t, \qquad \rho = 0.85, \tag{24}$$

whose additive shock term (`rffintay_aerr`) implements monetary-policy shocks. Section 6 exploits the switch structure directly, re-running the fiscal experiment under `dmpex` with `rfffix` set to the baseline funds-rate path.

3.11 Fiscal closure

Fiscal policy is closed by reaction functions on the *trend* personal tax rate (`trpt`), selected by switches: exogenous trend rates (`dfpex`), stabilisation of the debt-to-GDP ratio at a target (`dfpdbt`), or stabilisation of the surplus-to-GDP ratio (`dfpsrp`), the latter as coded:

$$\texttt{trpt}_t \;=\; \texttt{trpt}_{t-1} \;+\; \lambda\left[\tfrac{\texttt{gfsrpn}_{t-1}}{\texttt{xgdpn}_{t-1}} - \left(\texttt{gfsrt}_{t-1} + 0.0075\,\tilde{x}_{t-1}\right)\right], \tag{25}$$

so the trend tax rate drifts up (down) when the cyclically adjusted surplus ratio is below (above) target. The actual average personal tax rate (`trp`) then error-corrects to trend with a small

Table 2: Monetary policy rules in model.xml (April 2026 vintage)

Switch	Rule	Specification (coefficients as published)
dmpintay	Inertial Taylor (rffintay)	$r_t = 0.85\,r_{t-1} + 0.15\big[r^*_t + \bar{\pi}_t + 0.5\,(\bar{\pi}_t - \pi^{targ}) + 1.0\,\tilde{x}_t\big]$
dmptay	Taylor, output gap (rfftay)	$r_t = r^*_t + \bar{\pi}_t + 0.5\,(\bar{\pi}_t - \pi^{targ}) + 1.0\,\tilde{x}_t$
dmptlr	Taylor, unemp. gap (rfftlr)	$r_t = r^*_t - 0.5\,\pi^{targ} + 0.375\sum_{i=0}^{3}\pi_{t-i} + 1.1\,(\mathtt{lurnat}_t - \mathtt{lur}_t)$
dmpalt	Estimated rule (rffalt)	$r_t = 0.055 + 1.20\,r_{t-1} - 0.39\,r_{t-2} + 0.695\,\tilde{x}_t - 0.517\,\tilde{x}_{t-1} + 0.329\,\bar{\pi}_t$
dmpex	Exogenous nominal rate	$r_t = \mathtt{rfffix}_t$ (pre-set path)
dmprr	Exogenous real rate	$r_t = \mathtt{rrfix}_t + \bar{\pi}_t$
	Lower bound	$\mathtt{rff}_t \geq \mathtt{rffmin}_t$; threshold liftoff via dmptrsh, lurtrsh, pitrsh

The baseline LONGBASE projection sets dmpintay $= 1$ (all other switches 0), $\pi^{targ} = 2$, $r^* = 1.1$. The estimated rule rffalt has funds-rate lag coefficients summing to 0.81 and hence a long-run inflation response of $0.329/(1 - 0.81) \approx 1.7$.

procyclical term, and personal tax receipts follow the identity $\mathtt{tpn} = \mathtt{trp}\,(\mathtt{ypn} - \mathtt{gtn})$. All experiments in this paper use the Fed's standard demo configuration (surplus-ratio targeting, dfpdbt $= 0$, dfpsrp $= 1$); the tax experiment of Section 6 operates directly on trp.

4 Implementation

The package (frbus) is roughly 700 lines of Python over numpy, scipy, sympy and pandas, organised as a pipeline from the Fed's published artefacts to a compiled per-period solver. The public API mirrors the essentials of the Fed's pyfrbus: Frbus(path), .init_trac(start, end, data), .solve(start, end, data) and .exogenize([...]), so that the Fed's demo programs translate line-for-line.

4.1 The XML model schema

model.xml is a flat list of <variable> elements, one per model variable. Each carries a <name>, an <equation_type> (Behavioral, Identity or Exogenous), a <sector> tag (the basis of Table 1), free-text <definition> and <description> fields, and — for endogenous variables — one or more equation blocks. The <standard_equation> block contains the VAR-expectations equation in two redundant syntaxes (<eviews_equation> and <python_equation>), a list of <coeff> elements pairing coefficient names such as y_rffintay(3) with numeric values, and a <rhs_eq_var> list naming the endogenous variables appearing on the right-hand side; MCE variants, where they exist, live in parallel <mce_equation> blocks, which this implementation deliberately ignores. parse.py reads the file with the standard-library XML parser into a ModelSpec: variable names, endogenous/exogenous classification, per-equation coefficient name–value maps, and the raw EViews equation text. Nothing else is consulted — the XML file is the single source of truth for the model's semantics, and the LONGBASE database (data.py, loaded onto a quarterly pandas PeriodIndex; here the April 2026 vintage, 857 quarters from 1962Q1 into the Board's long-run projection) for its numbers.

4.2 Lexing and index substitution

`equations.py` and `lexing.py` normalise each equation in stages. EViews constructs are rewritten: difference operators `d(log(x),0,1)` become explicit lag expressions, lag notation `x(-2)` becomes an indexed reference, and coefficient references `y_eq(i)` are substituted by their numeric values from the `<coeff>` table. Every equation is flipped to residual form $F_i(\cdot) = 0$ and augmented with its additive shock term (`<endo>_aerr`, present in the published equations) and an additive tracking residual (`<endo>_trac`, added by the implementation for add-factoring). The lexer then tokenises each equation and performs *index substitution*: a current-period endogenous variable becomes an element `x[i]` of the unknown vector, and everything else — lags of endogenous variables, exogenous variables, shocks and add factors — becomes a reference `data[p, j]` into the (periods × variables) data matrix, with `p` the row offset implied by the lag. The result is a list of plain Python expression strings over exactly two array names, which is what makes the next stage possible. Because substitution is against a specific DataFrame column layout, the compiled artefacts are cached per layout and invalidated when `exogenize` changes the endogenous set (exogenised equations are simply dropped from the system and their variables re-classed as data).

4.3 Symbolic differentiation and the sparse Jacobian

`symbolic.py` evaluates the substituted equation strings in a `sympy` namespace in which `x` and `data` are container objects returning symbols literally named `x[i]` and `data[p,j]`; the string form of any `sympy` derivative is therefore directly executable Python again. The analytic Jacobian is built by differentiating each residual F_i with respect to each unknown x_j that appears in it, stored as a triplet list $(i, j, \partial F_i/\partial x_j)$ with the diagonal structurally present, as in `pyfrbus`. Non-smooth primitives (`max`, `min`, and the `@recode` indicator used by the policy-threshold equations) are given exact piecewise derivatives. `runtime.py` compiles the residual vector and each Jacobian entry into ordinary Python lambdas evaluated with `numpy`, assembling a `scipy` CSR matrix per evaluation.

The system is extremely sparse. For the standard configuration (284 equations), the symbolic Jacobian has **900 structural non-zeros** out of $284^2 = 80{,}656$ possible entries — a density of **1.12 per cent**, or a mean of 3.17 unknowns per equation (median 3). The densest rows have 11 entries (final sales `xfs` and the permanent-income expectations `zyht`, `zyhp`); 46 equations reference only their own variable contemporaneously, i.e. are fully recursive given the rest of the period's solution. This sparsity is inherited from the model's structure: most simultaneity flows through a handful of aggregates (output, the funds rate, inflation) while long chains of identities and lag-driven equations hang off them.

4.4 Structural block analysis and comparison with pyfrbus

Applying a strongly-connected-components decomposition to the 900-edge within-period dependency graph yields **162 components: one large simultaneous block of 120 equations, one of 3, one of 2, and 159 singletons**. This is precisely the structure the vendor's `pyfrbus` exploits: its `block_ordering` module topologically sorts the components and solves them in sequence — trivially for the recursive singletons, with Newton's method for the simultaneous cores — which shrinks each linear solve at the cost of bookkeeping (renumbering unknowns per block, threading solved values into later blocks). This implementation makes the opposite engineering choice: it solves the full 284-equation system in a single damped Newton loop per period. The single-block approach is simpler — no graph machinery, one compiled residual/Jacobian pair, and `exogenize` reduces to a row/column deletion — and at FRB/US's scale it is not a bottleneck: a sparse LU factorisation of a 284×284 matrix with 900 non-zeros costs well under a millisecond, so the recursive equations ride along in the Newton solve at negligible cost. Both approaches solve the same $F(x) = 0$; Section 5 confirms agreement to solver tolerance. For much larger systems — or the MCE stacked-time problem, where all quarters' equations are solved jointly — block decomposition and orderings would begin to pay.

4.5 The per-period damped Newton solver

Because the VAR model is backward-looking across quarters, `solver.py` solves period by period, using the previous quarter's solution as the initial guess. Each quarter the full simultaneous system $F(x) = 0$ is solved by a damped Newton method:

1. evaluate F and the sparse Jacobian J at the current iterate;
2. row-scale the system by $\text{diag}(1/\max_j |J_{ij}|)$ for conditioning;
3. factor the scaled Jacobian with a sparse LU decomposition (`scipy.sparse.linalg.splu`, i.e. SuperLU) and solve for the Newton step;
4. backtrack: halve the step until the model evaluates cleanly at the trial point — no domain errors (logs of non-positive arguments) and no NaNs in F or J;
5. iterate until the step norm falls below `xtol` $= 10^{-8}$, then require the residual norm to be below `rtol` $= 5 \times 10^{-4}$.

The damping criterion is deliberately permissive — it guards against leaving the domain of definition rather than enforcing monotone residual descent — which matches the vendor's behaviour and is sufficient in practice: starting from the previous quarter's solution, baseline and shocked quarters alike converge in a handful of iterations, and no experiment in this paper required anything beyond the default settings. Failures (singular Jacobian, exhausted backtracking) raise typed exceptions rather than returning silently.

4.6 Performance

Table 3 reports wall-clock times on the development machine (Apple Silicon laptop, single-threaded), measured on fresh runs of the experiment script in `figures/`. One-off model construction — XML parsing, lexing, symbolic differentiation of all 284 equations and compilation — takes about 0.7 seconds; after that, solving costs about **4.5 milliseconds per quarter**, so a 20-quarter policy simulation runs in under a tenth of a second and the full experiment suite of Section 6, spanning three model configurations and 40-quarter horizons, completes in a few seconds. Add-factoring (`init_trac`) is a single equation evaluation per quarter and is essentially free. These times remove any computational barrier to embedding FRB/US in bootstrap loops, optimisation over rule coefficients, or microsimulation pipelines.

4.7 Add factors

`init_trac` implements FRB/US's standard add-factoring ("tracking") mechanism: for each quarter in the simulation window it evaluates every equation at the baseline data and stores the

Table 3: Performance benchmarks (fresh runs, single core)

Operation	Time
Parse `model.xml`	0.012 s
Lex, differentiate and compile (one-off, incl. `init_trac` over 40q)	0.68 s
Solve, 20 quarters	0.088 s
Solve, 40 quarters	0.181 s
Per solved quarter	≈4.5 ms

Sparse LU via SuperLU; Jacobian: 900 non-zeros, density 1.12%. Times from `figures/run_experiments.py` on the April 2026 artefacts.

negated residual in the equation's `_trac` variable, so that the baseline is an exact solution of the adjusted model. Shocks are then imposed on top of the add-factored baseline (e.g. via `_aerr` terms), and simulated paths are read as deviations from base. Section 5 shows this reproduction is exact to machine precision.

4.8 Differences from pyfrbus

The implementation follows `pyfrbus` semantics exactly but departs from it in engineering terms:

- **Modern dependencies.** No pin of `sympy` 1.3, no `scikit-umfpack`/`symengine` requirement, no pre-2.0 `pandas`; the package installs on current Python (3.10+) with wheels only. Sparse solves use `scipy`'s bundled SuperLU rather than UMFPACK.
- **Single-block Newton.** `pyfrbus` decomposes each period's system into recursive and simultaneous blocks (Section 4.4) and solves them in sequence; this implementation solves the full simultaneous system in one Newton iteration loop. Both solve the same $F(x) = 0$, and Section 5 confirms the solutions agree to solver tolerance.
- **Scope.** MCE (rational-expectations) equation variants are not implemented — `Frbus(path, mce=...)` raises `NotImplementedError` — and the `mcontrol` trajectory-matching utility and stochastic simulations are likewise out of scope. The VAR model, add-factoring, exogenisation and shock simulation are complete.

5 Validation

The implementation is validated against the Fed's own artefacts in two layers, both of which run as hard gates in the package's test suite and continuous integration. All experiments use the VAR model over 2026Q1–2030Q4 with the Fed's standard demo fiscal configuration (surplus-ratio targeting: `dfpdbt` $= 0$, `dfpsrp` $= 1$) and the April 2026 LONGBASE vintage.

5.1 Tracking invariant

After `init_trac`, solving the baseline must reproduce LONGBASE exactly for all 284 endogenous variables in all 20 quarters. This exercises the entire pipeline — parser, lexer, coefficient substitution, compiled residuals, Jacobian and Newton solver — against the Fed's own data. Table 4 shows the result: the maximum absolute error across all 284×20 variable-quarters is 5.6×10^{-17}, i.e. machine precision.

Table 4: Tracking invariant: solved baseline vs. LONGBASE, 2026Q1–2030Q4

Metric	Value	Test gate
Max absolute error, all 284 endogenous × 20 quarters	5.6×10^{-17}	$< 10^{-8}$
Max relative error	5.6×10^{-17}	$< 10^{-10}$

5.2 Cross-validation against the Fed's pyfrbus

The identical experiment — `init_trac` followed by a 100-basis-point shock to `rffintay_aerr` in 2026Q1, mirroring the Fed's `demos/example1.py` — was run in both this implementation and the Fed's `pyfrbus`, and all 284 endogenous variables were compared over all 20 quarters. The vendor reference was produced by `pyfrbus` running in a separate Python 3.11 environment with its Newton solver at `xtol` $= 10^{-8}$; a documented script regenerates it, recording the out-of-repo patches needed to run the vendor package on modern dependencies (an `splu` shim replacing `scikit-umfpack`, unpinning `sympy`, a fix to the vendor's `Max/Min` derivative fallback under modern `symengine`, and `pandas` < 2).

Table 5: Cross-validation vs. pyfrbus: identical 100bp funds-rate shock, 2026Q1–2030Q4

Metric	Value	Test gate
Max absolute difference, all 284 endogenous × 20 quarters	6.0×10^{-9}	$< 10^{-6}$
Max relative difference (denominator clipped at 10^{-8})	4.9×10^{-8}	$< 10^{-6}$

Table 6: Cross-validation: agreement on key variables (max over 20 quarters)

Variable	Description	Max abs. diff.	Max rel. diff.
`xgdp`	Real GDP	1.5×10^{-10}	5.6×10^{-15}
`pcpi`	CPI price level	8.5×10^{-13}	2.5×10^{-15}
`picxfe`	Core PCE inflation	2.3×10^{-13}	1.2×10^{-13}
`lur`	Unemployment rate	4.0×10^{-13}	9.2×10^{-14}
`rff`	Federal funds rate	3.0×10^{-13}	1.1×10^{-13}

Tables 5 and 6 summarise the agreement. The worst-agreeing variables are near-zero expectational gap terms (`zgap05`, relative difference 4.9×10^{-8} on values of order 10^{-4}), which reflect the two solvers' respective step tolerances rather than any semantic difference; with the vendor solver at its default looser tolerance (`scipy.optimize.root`, `xtol` $= 10^{-6}$) the discrepancy is $\sim 1.7 \times 10^{-5}$ and again concentrated in the same terms. The headline variables agree to 10^{-10} or better in absolute terms. Since the vendor's block-decomposed solver and this implementation's single-block Newton solve the same $F(x) = 0$, agreement up to solver tolerance is the expected — and observed — outcome.

5.3 How large is 10^{-8}? The reference implementation against itself

A residual is only meaningful against a yardstick, and the natural one is the reference implementation's own version-to-version variation. The vendored reference is `pyfrbus` 1.0.0 (files dated

6 April 2022); the Board also ships 1.1.1 (5 February 2026), whose `model.xml` and LONGBASE are byte-identical to 1.0.0's, so a comparison between them isolates solver differences from model differences. Version 1.1.1 rewrites the Newton routine to reuse its LU factorisation across iterations, recomputing only when the residual fails to halve — a change to the iterate path, not to the fixed point. Repeating the identical shock experiment across all three implementations gives Table 7.

Table 7: Three-way comparison: identical 100bp funds-rate shock, 284 endogenous × 20 quarters

Comparison	Max abs. diff.	Max rel. diff.
This implementation vs. `pyfrbus` 1.0.0	6.0×10^{-9}	4.9×10^{-8}
This implementation vs. `pyfrbus` 1.1.1	1.4×10^{-8}	1.8×10^{-7}
`pyfrbus` 1.1.1 vs. `pyfrbus` 1.0.0	1.3×10^{-8}	1.3×10^{-7}

The third row is the informative one: the Board's own two releases differ from each other by as much as this implementation differs from either, and all three residuals are concentrated in the same near-zero series (`wpsn`, `zgap05`). The agreement documented above is therefore not an artefact of the particular vendored release — it sits at the scale of the reference implementation's own numerical noise. On the tracking invariant of Section 5.1, `pyfrbus` 1.1.1 reproduces LONGBASE to 1.1×10^{-8}, against this implementation's 5.6×10^{-17}. The committed reference remains 1.0.0 so that the regression gate has a fixed anchor. Only the VAR path was exercised in this comparison; 1.1.1 additionally fixes a lead-substitution guard that affects the MCE path, which this implementation does not support and which was therefore not tested.

6 Simulation results

This section reports six experiments: (i) the canonical 100-basis-point monetary-policy shock; (ii) the same shock followed over a 40-quarter horizon to trace the return to baseline; (iii) a government-purchases increase of 1 per cent of GDP under the inertial Taylor rule; (iv) the same purchases increase with the funds rate held fixed at its baseline path, isolating the role of the monetary response; (v) a personal-tax-rate cut of equal ex-ante budgetary size, giving the model's tax multiplier; and (vi) a cost-push price shock of the kind the 2025–2026 tariff debate has put back on the agenda. All experiments start from the add-factored April 2026 LONGBASE baseline with the demo fiscal configuration (surplus-ratio targeting), and all are regenerated by a single documented script (`figures/run_experiments.py`); the whole suite runs in a few seconds (Table 3).

6.1 A 100-basis-point monetary policy shock

The canonical FRB/US experiment adds 100 basis points to the additive shock term of the inertial Taylor rule (equation 24) in the first simulation quarter, on top of the add-factored baseline. Table 8 reports the full impulse responses over the first 20 quarters, as deviations from baseline.

The funds rate rises by exactly 100 basis points on impact and then decays as the inertial rule unwinds the shock, undershooting mildly once activity and inflation have fallen. Real GDP declines with a hump shape to a trough of −0.55 per cent seven quarters after the shock

Table 8: Impulse responses to a 100bp shock to the inertial Taylor rule (deviations from baseline)

Quarter	`rff` (pp)	`xgdp` (%)	`lur` (pp)	`picxfe` (pp)
2026Q1	1.000	0.001	−0.000	0.000
2026Q2	0.822	−0.187	0.099	−0.010
2026Q3	0.654	−0.290	0.151	−0.021
2026Q4	0.490	−0.436	0.210	−0.025
2027Q1	0.342	−0.483	0.227	−0.030
2027Q2	0.211	−0.528	0.246	−0.032
2027Q3	0.097	−0.544	0.254	−0.033
2027Q4	0.001	**−0.551**	**0.258**	**−0.034**
2028Q1	−0.079	−0.544	0.257	−0.033
2028Q2	−0.143	−0.526	0.250	−0.033
2028Q3	−0.191	−0.499	0.240	−0.032
2028Q4	−0.226	−0.464	0.226	−0.031
2029Q1	−0.248	−0.426	0.209	−0.030
2029Q2	−0.260	−0.384	0.190	−0.030
2029Q3	−0.262	−0.340	0.168	−0.029
2029Q4	−0.257	−0.296	0.146	−0.028
2030Q1	−0.245	−0.254	0.124	−0.027
2030Q2	−0.229	−0.214	0.101	−0.027
2030Q3	−0.209	−0.176	0.080	−0.026
2030Q4	−0.187	−0.142	0.059	−0.026

Bold entries mark the trough/peak responses. Source: fresh run of the released monetary-shock example with the April 2026 LONGBASE vintage; identical to the cross-validated experiment of Table 5.

(2027Q4), the unemployment rate peaks +0.26 percentage point above baseline in the same quarter, and core PCE inflation bottoms at −0.034 percentage point — the model's well-known flat reduced-form Phillips curve at work.

6.2 The long horizon: return to baseline over 40 quarters

Figure 1 extends the same experiment to 2035Q4 — forty quarters. The endogenous unwinding is slow but complete: after the 2027Q4 trough, the funds-rate undershoot (bottoming near −0.26 percentage point in 2029) gradually re-stimulates demand, output crosses to within a tenth of a per cent of baseline during 2031 and ends the horizon at −0.02 per cent, with unemployment slightly *below* baseline (−0.03 percentage point) as the recovery overshoots the employment gap. Inflation, by contrast, remains persistently below baseline throughout (−0.013 percentage point in 2035Q4): with a near-flat Phillips curve and slow-moving trend expectations, the price-level loss from the shock is never made up under the inertial Taylor rule — an inflation-targeting, not price-level-targeting, property that the makeup-strategy literature using FRB/US addresses directly (Bernanke et al., 2019). The 40-quarter horizon confirms that the VAR-expectations model is dynamically stable around the baseline: all reported deviations decay toward zero without oscillation beyond the single policy-induced undershoot cycle.

6.3 Comparison with published FRB/US simulations

Brayton et al. (2014) report the effects of an unanticipated 1-percentage-point increase in the federal funds rate in FRB/US under both expectations options. Under VAR expectations —

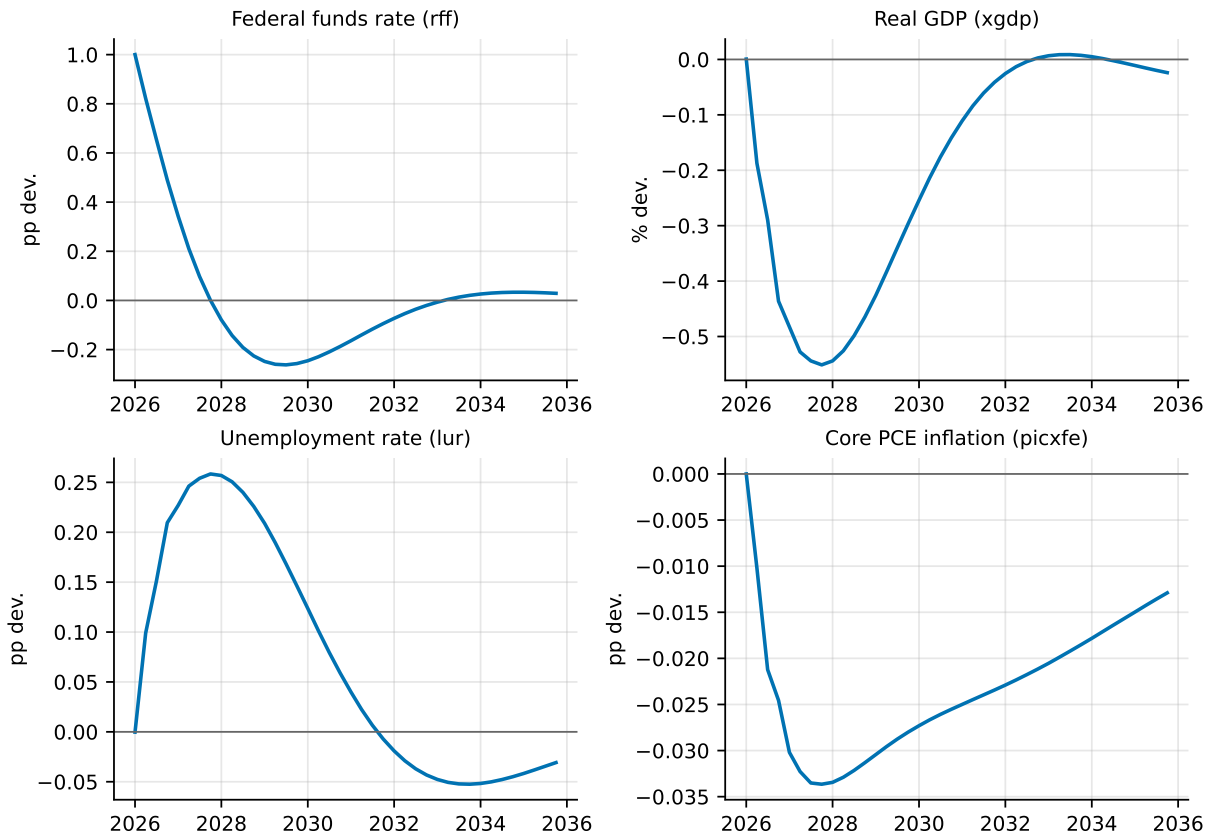

Figure 1: Responses to a 100bp inertial-Taylor-rule shock over 40 quarters (deviations from the add-factored baseline, 2026Q1–2035Q4).

the relevant comparison — they show the output gap declining by a maximum of about 0.4 percentage point, inflation falling by up to about 0.08 percentage point, and activity remaining below baseline for five to six years, with the real funds rate elevated for two to three years; under MCE the same shock moves the output gap only about −0.1 percentage point. Our responses sit squarely in this range: a peak output loss of half a per cent about two years out, unemployment up a quarter point, and a modest inflation decline, with the economy still below baseline at the five-year horizon. Exact magnitudes are not expected to coincide: the published figures reflect an earlier model vintage and baseline (2014 versus the April 2026 `model.xml` and LONGBASE), a different design of the funds-rate perturbation, and a decade of re-estimation — the current vintage's inflation response, in particular, is somewhat smaller than the 2014 figure, consistent with the further flattening of the estimated price block. The qualitative fingerprint of FRB/US under VAR expectations — hump-shaped and persistent real responses with small inflation movements — is exactly reproduced, and the quantitative agreement with the Fed's own *code* on the identical vintage is at the 10^{-9} level (Section 5).

6.4 A government-purchases experiment

As the baseline fiscal experiment, we exogenise real federal government purchases (`egfe`) and raise them by 1 per cent of baseline real GDP for eight quarters (2026Q1–2027Q4), leaving monetary policy on the inertial Taylor rule and fiscal closure on surplus-ratio targeting. Table 9 reports the results from a fresh run.

Table 9: Government purchases +1% of GDP for 8 quarters: selected responses

Quarter	Shock (% of GDP)	`xgdp` (%)	`lur` (pp)	`rff` (pp)
2026Q1	1.0	0.74	−0.31	0.11
2026Q4	1.0	0.69	−0.31	0.34
2027Q4	1.0	0.59	−0.31	0.48
2028Q4	0.0	−0.30	0.10	0.12
2030Q4	0.0	−0.33	0.18	−0.21

Deviations from baseline. The implied spending multiplier is 0.74 on impact and averages about 0.72 over the first year, declining as the funds rate rises; output falls below baseline after the spending expires.

The impact multiplier is 0.74, averaging roughly 0.7 over the first year and eroding to 0.59 by the end of the second year as the inertial Taylor rule raises the funds rate by nearly 50 basis points and crowds out private demand; once the stimulus expires, output dips below baseline. The experiment also illustrates the platform's `exogenize` mechanism, which converts listed endogenous variables into exogenous instruments while keeping the remaining system simultaneous.

6.5 The same shock under a fixed funds rate: monetary accommodation matters

How much of the multiplier erosion is the central bank's doing? FRB/US's switch architecture (equation 23) makes the counterfactual a two-line change: we set `dmpintay` $= 0$, `dmpex` $= 1$ and fix `rfffix` at the baseline funds-rate path, so the nominal rate simply does not respond to the fiscal expansion, and re-run the identical purchases shock. Table 10 and Figure 2 compare the two regimes.

Table 10: Government purchases +1% of GDP: inertial Taylor rule versus fixed funds rate

	Inertial Taylor rule			Fixed nominal funds rate		
Quarter	`xgdp` (%)	`rff` (pp)	`picxfe` (pp)	`xgdp` (%)	`rff` (pp)	`picxfe` (pp)
2026Q1	0.74	0.11	0.00	0.74	−0.07	0.00
2026Q4	0.69	0.34	0.02	0.84	−0.12	0.03
2027Q2	0.67	0.43	0.02	0.97	−0.08	0.04
2027Q4	0.59	0.48	0.02	1.06	−0.06	0.05
2028Q4	−0.30	0.12	0.01	0.37	0.02	0.05
2030Q4	−0.33	−0.21	0.01	0.17	0.01	0.05
Year-1 multiplier		0.72			0.79	
Year-2 multiplier		0.64			0.99	

Deviations from baseline; multipliers are calendar-year averages of the GDP response per unit of shock. Under `dmpex` the funds rate is pegged to its baseline path (small residual movements in `rff` reflect the effective-lower-bound recode operating on the baseline path); long rates still move through expected *future* rates in the VAR expectations block.

The two regimes start identically — the inertial rule moves only 11 basis points in the impact quarter, so the impact multiplier is 0.74 either way — and then diverge sharply. Under the Taylor rule the cumulated 48-basis-point tightening caps the output gain and produces the post-expiry undershoot; with the rate fixed, the multiplier *grows* over the spending window, averaging 0.99 in year two and peaking at 1.06, as rising inflation lowers real rates and no crowding-out force operates. The inflation response is correspondingly larger and far more persistent (+0.05 percentage point and still there in 2030, versus +0.02 fading to zero under the rule). This reproduces the central finding the Board reports for FRB/US and that the multi-model comparison of Coenen et al. (2012) documents across U.S. models: spending multipliers of roughly one when the funds rate is held fixed — FRB/US's first-year multiplier is 0.7–1.0 without accommodation, rising to about 1.1–1.2 with two years of accommodation — and noticeably below one when monetary policy leans against the stimulus.

6.6 A personal-tax-cut experiment

The natural PolicyEngine question is the tax counterpart. We exogenise the average personal tax rate (`trp`) and cut it for eight quarters by exactly the amount that reduces personal tax receipts by 1 per cent of GDP ex ante, i.e. $\Delta \texttt{trp}_t = -0.01\,\texttt{xgdpn}_t/(\texttt{ypn}_t - \texttt{gtn}_t)$ evaluated on the baseline — about 1.4 percentage points on the average rate. The realised revenue loss is 0.96–0.99 per cent of GDP throughout the window (the small shortfall reflecting the induced rise in incomes), so the experiment is budget-comparable with the purchases shock. Monetary policy stays on the inertial Taylor rule.

The tax multiplier is far smaller and more delayed than the spending multiplier: 0.17 on impact, an average of 0.22 in the first year and 0.32 in the second, peaking at 0.34 in the final quarter of the cut (Table 11, Figure 3). The economics follows directly from the consumption block of Section 3.6: only the rule-of-thumb 31 per cent of consumers respond to current after-tax income one-for-one, while optimising households spread a known-temporary two-year tax cut over their (25-per-cent-discounted) permanent income, and much of the impact effect leaks

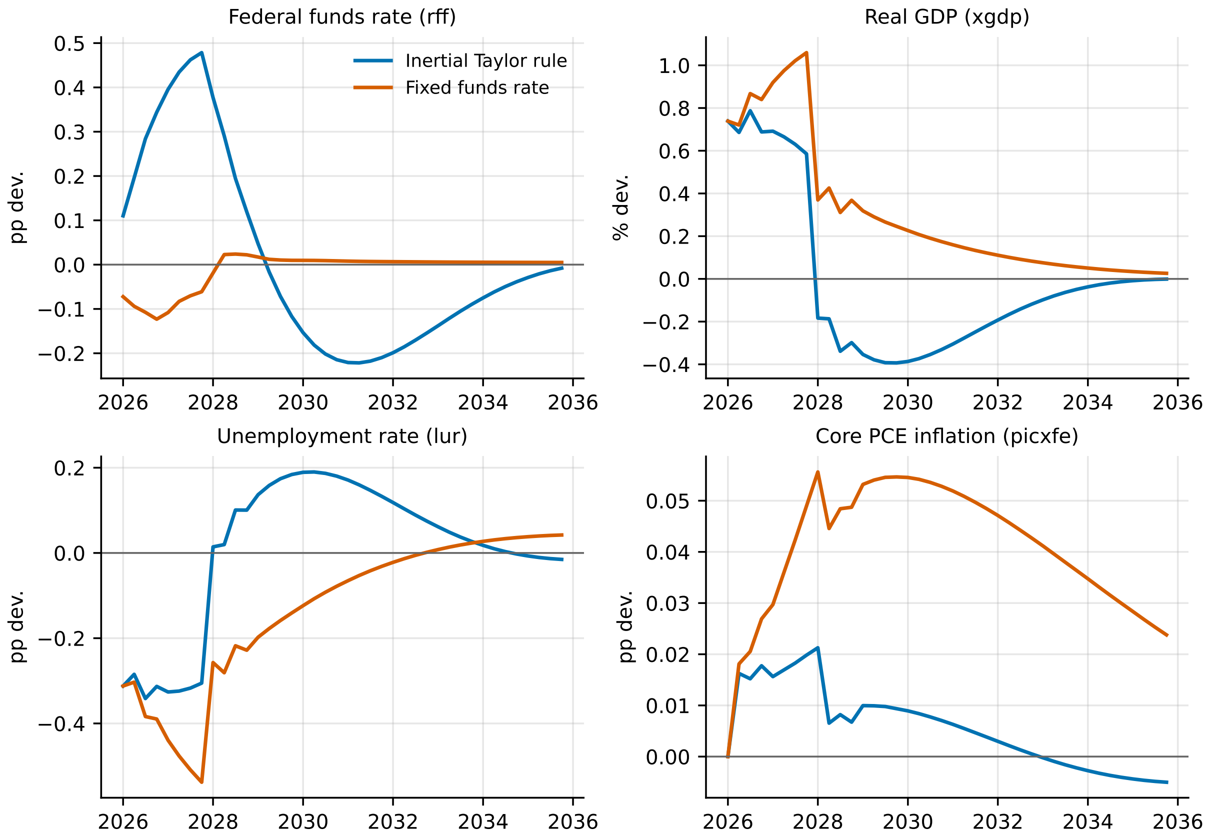

Figure 2: Government purchases +1% of GDP for eight quarters, under the inertial Taylor rule and under a fixed nominal funds rate.

Table 11: Personal tax cut of 1% of GDP for 8 quarters: selected responses

Quarter	Revenue change (% of GDP)	`xgdp` (%)	`lur` (pp)	`rff` (pp)	`picxfe` (pp)
2026Q1	−0.99	0.17	−0.07	0.02	0.00
2026Q2	−0.98	0.21	−0.09	0.05	0.00
2026Q4	−0.98	0.27	−0.12	0.11	0.01
2027Q2	−0.97	0.31	−0.15	0.16	0.01
2027Q4	−0.96	**0.34**	−0.17	0.21	0.01
2028Q4	0.04	0.10	−0.07	0.17	0.01
2030Q4	0.03	−0.04	0.01	0.05	0.01

Deviations from baseline. The tax rate `trp` is cut by $0.01 \cdot \texttt{xgdpn}/(\texttt{ypn} - \texttt{gtn})$, an ex-ante revenue loss of 1% of GDP; the realised loss (column 2) is computed from simulated `tpn`. After expiry the surplus-targeting closure claws back a small amount of revenue.

into saving and imports. The hump shape — the multiplier *rises* over the window as PAC error-correction pulls consumption toward its raised target — is the mirror image of the front-loaded purchases multiplier, which enters demand directly in quarter one.

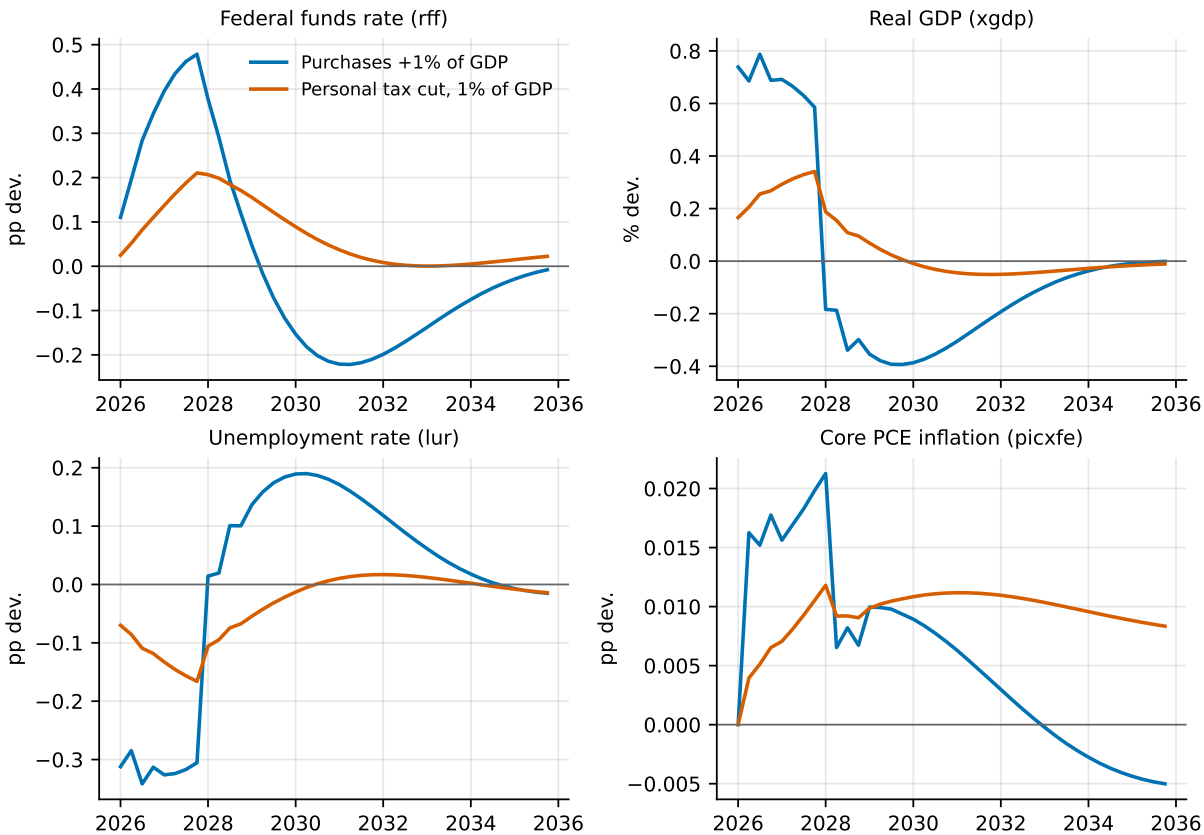

Figure 3: Equal-cost fiscal experiments under the inertial Taylor rule: government purchases versus a personal tax cut, each 1% of GDP for eight quarters.

6.7 A cost-push price shock

The 2025–2026 tariff rounds revived the question of how the model economy digests a supply-side price impulse. No published FRB/US treatment of a tariff scenario exists to replicate — the Board's recent tariff analyses use trade models, not FRB/US (Cuba-Borda et al., 2025) — so this experiment is offered as the platform's native answer to the generic question those analyses pose: what does a persistent cost-push impulse do under an estimated policy rule? We add 1 percentage point to the additive shock term of the core PCE inflation equation (`picxfe_aerr`) for four quarters, 2026Q1–2026Q4, leaving all policy settings at their defaults, and solve over 40 quarters.

Because the price block is indexed to its own lags (equation 14), the impulse compounds while it lasts: core inflation rises 1.0 percentage point on impact and peaks at +2.2 percentage points in 2026Q4 (a first-year average of +1.7), before decaying with a half-life of about a year once the shock is switched off. The inertial Taylor rule tightens progressively, the funds rate peaking +1.1 percentage points above baseline in 2027Q3 — *after* inflation has peaked, the inertia at work — and real GDP falls to a trough of −0.87 per cent in 2028Q3 with unemployment up 0.35 percentage point (Figure 4). Two model properties stand out. First, the output cost of the disinflation is borne late: activity troughs seven quarters after the inflation peak, when the price impulse itself has largely passed. Second, the same flat Phillips curve that mutes the inflation response to demand shocks works in reverse here: the induced slack claws back inflation only slowly, and core inflation still sits slightly *below* baseline (−0.09 percentage point) in 2030 as the accumulated tightening bites, and the unwinding produces one further mild activity cycle (output overshooting to +0.3 per cent around 2034) before the economy settles back. Qualitatively this matches the profile of the Board's trade-model tariff analysis — a persistent, roughly half-point inflation rise from a plausible tariff shock, with a persistent output drag (Cuba-Borda et al., 2025) — scaled up by our deliberately larger normalised impulse; a quantitative tariff pass-through mapping would require the trade-price channels those models supply.

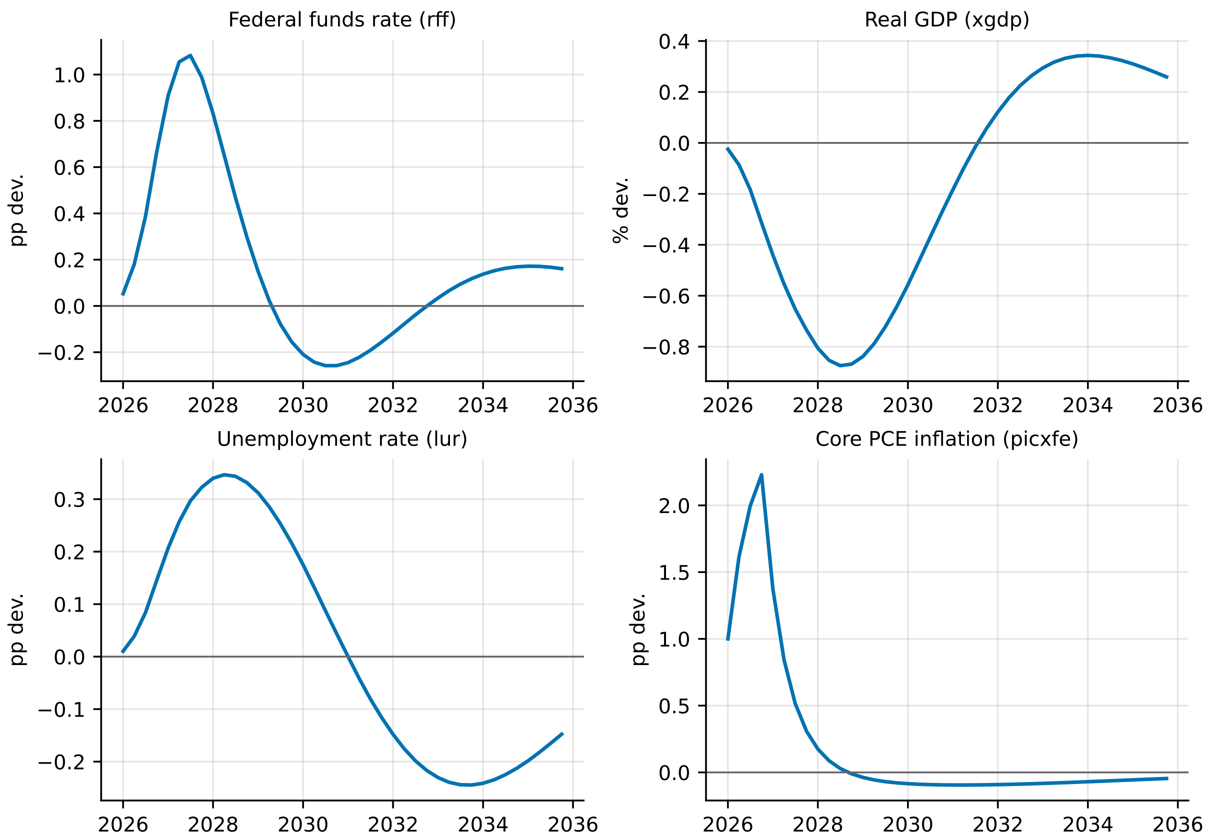

Figure 4: Cost-push shock: +1pp on the core PCE inflation equation's additive shock term for four quarters (2026Q1–2026Q4), under the inertial Taylor rule and surplus-ratio fiscal targeting.

6.8 The multiplier menu and published benchmarks

Table 12 collects the experiments into a multiplier menu and sets them against published figures. The pattern matches the Board's documented fiscal properties and the cross-model evidence closely: FRB/US spending multipliers are somewhat below one under an estimated policy rule and close to one with a pegged rate (Brayton et al., 2014; Coenen et al., 2012); personal tax multipliers are a fraction of spending multipliers — Coenen et al. (2012) report first-year labour-tax-cut multipliers of 0.2–0.4 across U.S. models, almost invariant to monetary accommodation, and CBO's central two-year multiplier for broad individual tax cuts is about 0.3 (range 0.1–0.6) (Whalen and Reichling, 2015), while the FRB/US-based fiscal-effect measure of Cashin et al. (2018) likewise embodies personal-tax MPCs of a few tenths. Our year-one figure of 0.22 and year-two figure of 0.32 sit inside every one of those ranges. Note what is *not* in the table: at the effective lower bound, published FRB/US analysis finds substantially larger fiscal multipliers because the accommodation is automatic (Reifschneider and Williams, 2000; Coenen et al., 2012); our fixed-rate experiment is the deterministic analogue of that mechanism.

Table 12: First- and second-year multipliers: this implementation versus published benchmarks

Experiment	Year 1	Year 2	Published benchmark
Purchases, inertial Taylor rule	0.72	0.64	0.7–1.0 first-year, no accommodation (Coenen et al., 2012)
Purchases, fixed funds rate	0.79	0.99	≈ 1 with rate held fixed (Brayton et al., 2014); 1.1–1.2 with 2-yr accommodation (Coenen et al., 2012)
Personal tax cut, Taylor rule	0.22	0.32	0.2–0.4 labour-tax cut (Coenen et al., 2012); ≈ 0.3 CBO central estimate (Whalen and Reichling, 2015)

Multipliers are calendar-year average GDP deviations per unit of ex-ante budgetary cost (1% of GDP). All shocks last eight quarters, 2026Q1–2027Q4.

7 Comparison with the reference implementation, the published record and other models

7.1 The Fed's pyfrbus and the published FRB/US record

This section consolidates the paper's headline numbers against the published record in one place. Two kinds of comparison must be kept apart. The first is *like-for-like*: the Fed's own `pyfrbus` reference implementation runs on this machine, on the same `model.xml`, the same LONGBASE vintage and the identical shock, so the two implementations should agree — and they do, to solver precision. Re-running the Board's demo experiment (a 100-basis-point add-factor to `rffintay` under surplus-ratio fiscal targeting) in both codes gives a maximum absolute difference of 6.0×10^{-9} across all 284 endogenous variables and all twenty quarters, i.e. solver-tolerance noise. The second kind of comparison is against *published* FRB/US output — most usefully the 2014 FEDS Note responses under VAR expectations (Brayton et al., 2014) and the cross-model fiscal ranges of Coenen et al. (2012) and Whalen and Reichling (2015). There the numbers can only be close, not identical: the published simulations were produced on earlier model and database vintages, from different baselines, and with shock designs (e.g. persistence of the funds-rate innovation) that the notes do not fully pin down. Table 13 shows both kinds side by side.

7.2 Recent Federal Reserve work with FRB/US

The published record did not stop in 2014, and the more recent Board literature provides two further, if less directly numerical, checks. First, Brayton and Reifschneider (2022) build their linearised LINVER model precisely around the experiment run here — a 100-basis-point shock to

Table 13: This implementation versus the Fed's pyfrbus and published FRB/US results

Quantity	Ours	Published	Deviation and reading
100bp funds-rate shock (VAR expectations)			
Real GDP, trough	−0.55% (yr 2)	—	FEDS Note reports the gap, not GDP
Output gap, trough	−0.50pp (yr 1.5)	≈ −0.4pp max	≈ 0.1pp deeper; vintage/shock design (Brayton et al., 2014)
Unemployment, peak	+0.26pp (yr 2)	rises (Okun-consistent)	consistent; peak ≈ half the gap trough
Core PCE inflation, trough	−0.034pp (yr 2)	≈ −0.08pp	0.05pp smaller; flatter current-vintage Phillips curve (Brayton et al., 2014)
vs. vendor pyfrbus (like-for-like)	identical to 6.0×10^{-9} (284 vars)		exact by construction (Board of Governors of the Federal Reserve System, 2026a)
Government purchases, 1% of GDP			
Multiplier, yr 1, Taylor rule	0.72	0.7–1.0 (no accommodation)	inside range (Coenen et al., 2012)
Multiplier, yr 2, fixed funds rate	0.99	"roughly one" pegged; 1.1–1.2 with 2-yr accommodation	matches the Board's characterisation; just below cross-model accommodation range (Brayton et al., 2014; Coenen et al., 2012)
Personal tax cut, 1% of GDP ex ante			
Multiplier, yrs 1–2	0.22 → 0.32	0.2–0.4; CBO central ≈ 0.3	inside both ranges (Coenen et al., 2012; Whalen and Reichling, 2015)

"Ours" entries are computed from `figures/results.json`. The pyfrbus row re-runs the Board's demo shock in the vendor package (relaxed dependency pins, Newton solver at `xtol`$= 10^{-8}$) and in this implementation on the identical baseline. Published 100bp responses are read from the 2014 FEDS Note charts (VAR expectations) and are therefore approximate.

the inertial Taylor rule — and report goodness-of-fit statistics of 0.999–1.000 between LINVER and full FRB/US for the output-gap, inflation and funds-rate responses to that shock; the impulse-response *shapes* they plot (hump-shaped activity decline, small inflation response) are the shapes in Figure 1, though the paper quantifies magnitudes only graphically. Second, the framework-review literature quantifies how much the expectations option matters, in a way that brackets our VAR-only results: Hebden et al. (2020) report that, relative to full model-consistent expectations, inflation expectations respond to policy news two years ahead by approximately zero under VAR expectations, 40 per cent with model-consistent financial markets only, and 80 per cent when wage–price setters are also model-consistent. Our implementation sits at the "approximately zero" end of that spectrum by construction, which is why forward-guidance-style exercises are out of scope (Section 8) and why our funds-rate-shock responses should be (and are) at the large end of the published range. Kiley (2025), the most recent Board research use of FRB/US at the time of writing, likewise runs its policy-rule comparisons across expectations settings rather than publishing a canonical funds-rate IRF; no 2024–2026 Board publication does, leaving the 2014 FEDS Note and LINVER as the reference points used above.

7.3 Situating FRB/US among other U.S. benchmarks

A different question from implementation fidelity is where FRB/US's responses sit relative to *other* models and to the reduced-form evidence. Table 14 collects benchmarks for the effect of a 100-basis-point policy-rate shock on activity. Three caveats govern the reading. The response variable differs (GDP here; industrial production, a far more cyclical series, in most of the VAR literature); the shock designs differ (our shock decays with the inertial rule, VAR shocks are typically one-off innovations); and the CBO row is deliberately a non-comparison — CBO's published interest-rate "rule of thumb" holds GDP fixed by design and prices out only the budgetary cost, so it cannot be read as a macro response at all.

The placement is comfortable: FRB/US under VAR expectations delivers activity effects at the moderate end of the empirical range — above the smallest CEE-style VAR estimates, close to the post-1984 FAVAR evidence once industrial production's greater cyclicality is taken into account, and well below the "medium effects" narrative-identification numbers — with a peak-effect timing (about two years) shared by nearly every entry. The NY Fed DSGE attributes a somewhat larger per-shock output effect but the same flat-Phillips-curve property of small inflation responses. On the fiscal side, the CBO comparison is already embedded in Table 13: CBO's published ranges are wide (0.5–2.5 for federal purchases, 0.3–1.5 for broad two-year tax cuts, with the central individual-tax multiplier around 0.3), and every multiplier in Table 12 lies inside them (Whalen and Reichling, 2015; Congressional Budget Office, 2022).

7.4 Reading the comparisons

The like-for-like agreement is the decisive test: on identical inputs the two codebases are the same model to solver precision, so the residual gaps against the 2014 FEDS Note — a slightly deeper output-gap trough and a roughly half-as-large inflation response — reflect model vintage and shock design, not implementation error. Both deltas move in the direction expected from documented vintage changes, most notably the progressive flattening of the estimated price Phillips curve between the 2014 public release and the current one. Figure 5 summarises the fiscal comparison graphically, Figure 6 overlays the published peak responses on our impulse responses, and Figure 7 shows the ours-versus-pyfrbus overlay, in which the two lines are visually

Table 14: Effect of a 100bp policy-rate shock on activity: FRB/US versus other U.S. benchmarks

Model / evidence	Activity response	Notes
This implementation (FRB/US, VAR expectations)	GDP −0.55%, trough at 8 quarters	unemployment +0.26pp; core inflation −0.034pp
FRB/US, MCE (Brayton et al., 2014)	output gap ≈ −0.1pp	same shock, model-consistent expectations
NY Fed DSGE (Del Negro et al., 2023)	output ≈ −1.25% below counterfactual at peak	cumulative effect of two ≈100bp shocks (2022Q4, 2023Q1); per shock roughly comparable to FRB/US; inflation response "small" (flat Phillips curve)
FAVAR, post-1984 sample (Ramey, 2016)	industrial production −0.7%, trough at 24 months	Boivin–Kiley–Mishkin estimates as tabulated by Ramey
CEE-style VAR (Ramey, 2016)	industrial production −0.2% to −0.5% at trough	sample-dependent; monetary shocks explain <7% of IP variance at 24 months
Romer–Romer narrative / proxy-SVAR (Ramey, 2016)	industrial production −1 to −3%, trough ≈18 months	the "medium effects" end of the evidence; one variant: unemployment +0.2pp at month 30
ECB-BASE (Angelini et al., 2019)	hump-shaped decline, stated to be "in the quantitative ballpark" of FRB/US	euro-area sibling of the FRB/US design; magnitudes shown only graphically
CBO rule of thumb (Congressional Budget Office, 2022)	none published (GDP held fixed)	+10bp on all rates ⇒ deficits +$285bn over 2023–32; budgetary costing only

"This implementation" from Table 8. VAR-literature entries are trough responses of industrial production to a shock normalised to a 100bp funds-rate movement, as surveyed in Ramey (2016); industrial production is roughly three times as cyclical as GDP, so these are not directly convertible into GDP units.

indistinguishable.

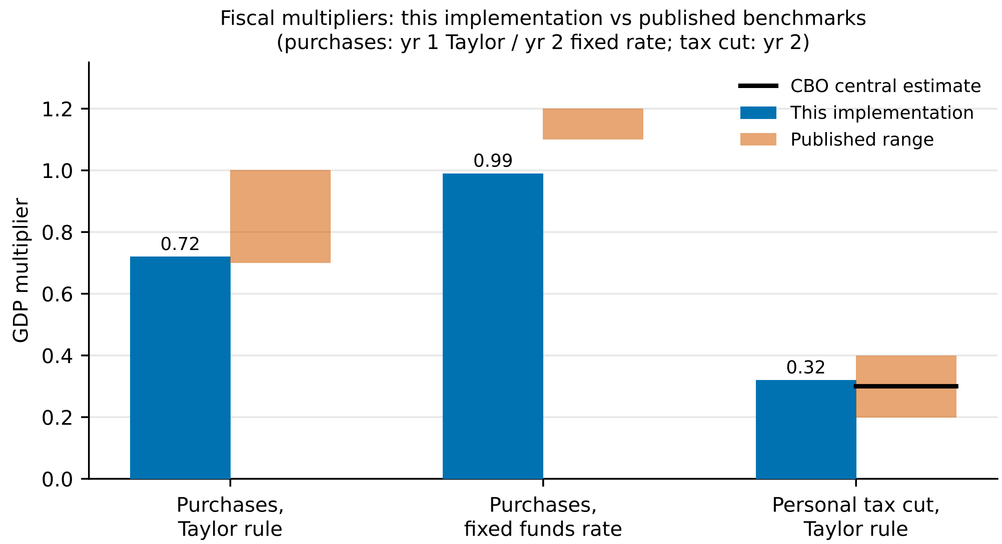

Figure 5: Fiscal multipliers: this implementation (bars) against the Coenen et al. (2012) cross-model ranges (bands) and the CBO central estimate (Whalen and Reichling, 2015).

8 Limitations and conclusion

Four limitations should frame any use of this implementation.

VAR expectations only. Only the backward-looking VAR expectations version of FRB/US is implemented. The MCE (rational-expectations) equation variants — under which agents' expectations are consistent with the model's own future paths, and which the Fed uses for exercises such as forward-guidance and threshold-policy analysis — raise `NotImplementedError`. As Section 6 notes, the two expectations options can deliver materially different quantitative answers (an output-gap response of -0.4 versus -0.1 percentage point to the same funds-rate shock in Brayton et al., 2014), so results from this package speak only to the VAR configuration. The `mcontrol` trajectory-matching utility is likewise not implemented.

Data vintage. All results are tied to the April 2026 LONGBASE vintage and the contemporaneous `model.xml`. The Board re-estimates equations and revises the projection database over time, and coefficients, baseline paths and hence impulse responses all shift with vintage, so users should refresh the vendored artefacts from the Board's website (Board of Governors of the Federal Reserve System, 2026a) and re-run the validation gates when doing so. One caution on reading those pages: as of 18 July 2026 the model-package page labels its data download "Updated: May 5, 2026", but the archive served from that link is byte-identical to the April vintage vendored here (same SHA-256; every member stamped 8 April 2026). The page label tracks the page, not necessarily the artefact.

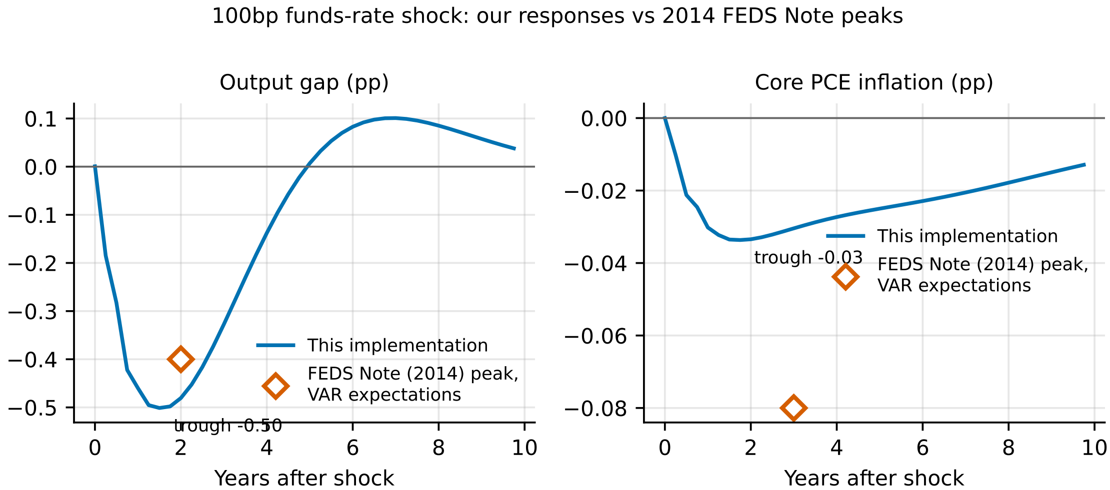

Figure 6: 100bp funds-rate shock: our output-gap and core-inflation responses with the 2014 FEDS Note peak responses under VAR expectations (Brayton et al., 2014) marked (diamonds; magnitudes read from the published charts, timing approximate).

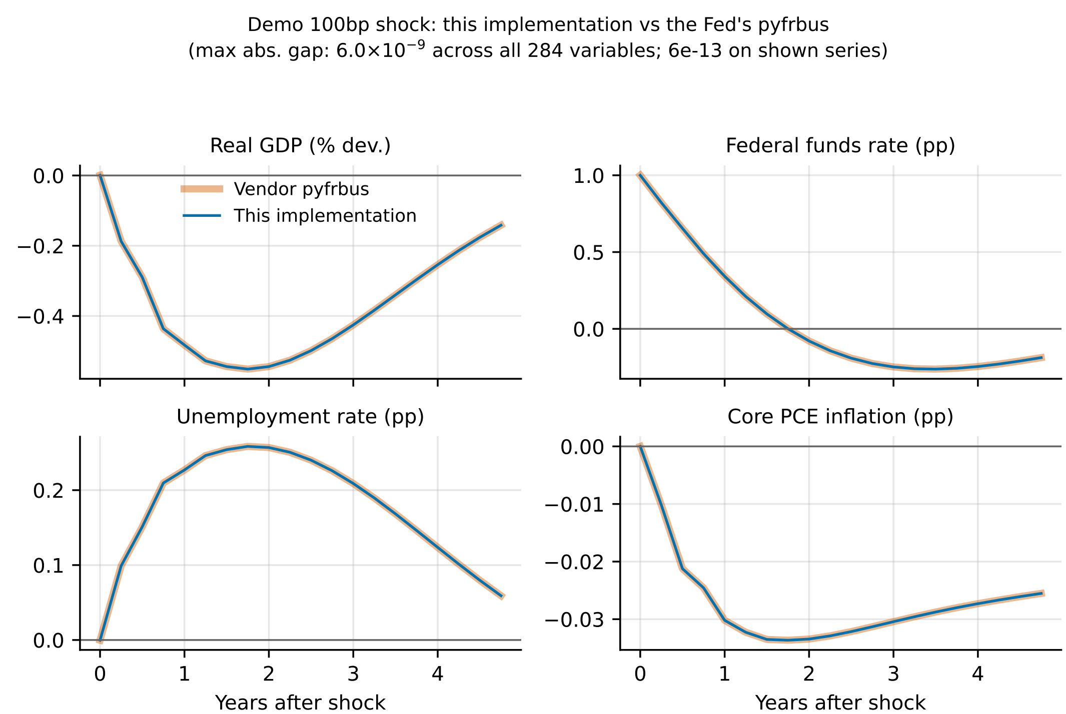

Figure 7: Like-for-like validation: the Board's demo 100bp shock run in this implementation and in the vendor pyfrbus package. The thick and thin lines coincide; the maximum absolute gap is 6.0×10^{-9} across all 284 endogenous variables (6×10^{-13} on the series shown).

Not an official product. FRB/US, its database and its documentation are produced and published by the Federal Reserve Board in the public domain; this reimplementation is an independent open-source project of PolicyEngine and is neither produced, endorsed, nor reviewed by the Board. Fidelity claims rest entirely on the reproducible validation evidence of Section 5.

Model-inherent caveats. The implementation inherits FRB/US's own modelling choices — estimated PAC dynamics, a flat reduced-form Phillips curve, policy closure by simple rules — and the usual caveats about using an estimated backward-looking system for counterfactuals far from the estimation sample apply with full force. The package now implements seeded joint historical-residual bootstrap simulations using the official `stochastic_type` inventory. Contemporaneous residual dependence is preserved and failed replications remain visible. The experiments in this paper remain deterministic, however, and no probability result is claimed without a separately reviewed residual window, policy closure, and convergence report. Model and data archives are independently hash-gated because the Board updates them on separate schedules.

Within those bounds, the contribution is a small, dependable, continuously tested platform: the Fed's public model definition and data, parsed and solved on a modern scientific-Python stack, reproducing the Fed's own reference implementation to 10^{-9} on identical experiments and its published simulation properties in sign, shape and magnitude. Within PolicyEngine Macro it provides the U.S. macroeconomic backbone against which microsimulation-based tax and transfer analyses can be given dynamic, model-consistent macro context, and a template — artefact parsing, symbolic Jacobians, add-factoring, hard validation gates in CI — for bringing other public-sector macro models into the open-source ecosystem.

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A Variable glossary

Table 15 defines every FRB/US mnemonic reported in the tables and figures of this paper, with definitions quoted from the `<definition>` fields of the April 2026 `model.xml`. Type "B" denotes a behavioural equation, "I" an identity, "X" an exogenous variable.

Table 15: Glossary of reported FRB/US variables

Mnemonic	Type	Definition
`xgdp`	B	GDP, chain-weighted 2012 dollars
`xgdpn`	I	GDP, current dollars
`xgap2`	I	Output gap for GDP, 100 (actual/potential − 1)
`rff`	B	Federal funds rate
`rffintay`	B	Funds rate implied by the inertial Taylor rule
`rfffix`	X	Funds rate given by a fixed, pre-determined path
`rffmin`	X	Minimum nominal funds rate (0 imposes the ZLB)
`rstar`	B	Equilibrium real funds rate perceived by policymakers
`pitarg`	X	Target rate of consumption price inflation
`picxfe`	B	Core PCE inflation (ex. food and energy, annual rate)
`pieci`	B	Growth of ECI hourly compensation (annual rate)
`lur`	B	Civilian unemployment rate (break adjusted)
`lurnat`	B	Natural rate of unemployment (random walk)
`lhp`	B	Aggregate labour hours, business sector
`leh`	I	Civilian employment (break adjusted)
`eco`	B	Consumer spending, non-durables and non-housing services
`ecd`	B	Consumer spending, durable goods
`eh`	B	Residential investment
`ebfi`	B	Business fixed investment
`egfe`	B	Federal government expenditures (CW 2012$), the purchases instrument
`trp`	B	Average personal income tax (and non-tax) rate, the tax instrument
`trpt`	B	Trend personal tax rate, moved by the fiscal reaction functions
`tpn`	I	Personal income tax receipts, current dollars (= `trp` (`ypn` − `gtn`))
`ypn`	I	Personal income
`gtn`	B	Federal government net transfer payments
`gfsrpn`	I	Federal government budget surplus
`rg10`	I	10-year Treasury bond rate
`rbbb`	I	S&P BBB corporate bond rate
`wpo`	I	Household property wealth ex. stock market, real
`wps`	I	Household stock-market wealth, real
`dmpintay`, `dmpex`	X	Monetary policy switches: inertial Taylor rule; exogenous funds rate
`dfpdbt`, `dfpsrp`	X	Fiscal closure switches: debt-ratio; surplus-ratio stabilisation
`*_aerr`	X	Additive shock term of each behavioural equation
`*_trac`	X	Additive tracking residual (add factor), set by `init_trac`

B Deterministic simulations: caveats

All experiments in this paper are *deterministic*: a single perturbed path is compared with a single add-factored baseline, with every stochastic term held at zero. Three consequences deserve emphasis.

No shock uncertainty. The reported responses are point paths, not distributions. Much of the published FRB/US policy literature — Reifschneider and Williams (2000), Reifschneider (2016), Bernanke et al. (2019) — rests on *stochastic* simulations in which historical residuals are drawn repeatedly and statistics such as the frequency of binding lower bounds are computed across thousands of paths. This implementation does not yet provide a stochastic-simulation driver, so questions of that type (how often does the ZLB bind? what is the variance of inflation under rule X?) are out of reach; the deterministic responses reported here are the impulse-response building blocks such exercises use.

Nonlinearity and state dependence. Because the model is nonlinear (the lower-bound recode, logs, the threshold switches), deterministic responses depend on the baseline around which they are computed, and the mean of stochastic paths need not equal the deterministic path. In particular, near the effective lower bound the deterministic economy responds asymmetrically to expansionary and contractionary shocks; the multipliers of Section 6 are computed from a baseline in which the funds rate is comfortably positive (around 3–4 per cent), and would differ at the ELB.

Certainty-equivalent expectations. Under VAR expectations agents' forecasts come from fixed-coefficient VARs and are unaffected by the experiment's announced future path — a deliberately boundedly rational assumption. Announced-in-advance policies, credibility effects and forward guidance require the MCE variants, which are not implemented (Section 8).

C Reproduction recipe

Every number and figure in this paper regenerates from public artefacts:

1. **Install.** Python 3.10+; `uv venv && uv pip install -e ".[dev]"` in the package repository installs `frbus` and its (unpinned, current) dependencies in a few seconds. The Fed artefacts — `model.xml`, `LONGBASE.TXT` (April 2026 vintage) and the reference `pyfrbus` — are vendored unmodified under `vendor/`, as published in the public domain (Board of Governors of the Federal Reserve System, 2026a).

2. **Validation gates.** `pytest` runs the tracking-invariant test (baseline reproduced to $< 10^{-8}$; observed 5.6×10^{-17}) and the cross-validation test against the stored `pyfrbus` reference (all 284 endogenous variables, 20 quarters, $< 10^{-6}$; observed 6.0×10^{-9}). `scripts/generate_vendor_reference.sh` documents how the vendor reference was produced in a separate patched environment.

3. **Experiments.** `figures/run_experiments.py` (alongside this paper's source) runs all six experiments of Section 6 — the 40-quarter monetary shock, the purchases shock under both monetary regimes, the tax cut and the cost-push shock — writing `results.json` with every reported series and the colourblind-safe PDF figures; `figures/make_comparison_figures.py` draws the comparison overlays of Section 7. Total runtime is a few seconds; the benchmark numbers of Table 3 are printed by the same script.

4. **Paper.** `latexmk -pdf main.tex` in the paper directory rebuilds this document, pulling tables' source numbers from the committed script output.

Because the Board revises `model.xml` and LONGBASE over time, reproductions on later vintages should expect coefficient and baseline differences; re-running the two validation gates after refreshing `vendor/` is the intended workflow.